

Precalculus - Complex Numbers and Complex Zeros - Answer Key

Common Core N-CN.A.1, N-CN.A.2, N-CN.A.3, N-CN.C.7, N-CN.C.8, N-CN.C.9 · maths-pdf.fr/en/

Use this key to check each complex-number problem. Every step shows where i squared becomes -1 , and final answers are in bold.

Precalculus · Common Core State Standards: **N-CN.A.1, N-CN.A.2, N-CN.A.3, N-CN.C.7, N-CN.C.8, N-CN.C.9**

Exercise 1 - Square roots of negatives

$$\sqrt{-36} = 6i. \sqrt{-20} = i\sqrt{20} = 2i\sqrt{5}.$$

$6i$ and $2i\sqrt{5}$

Exercise 2 - Powers of i

$$27 = 4 \cdot 6 + 3, \text{ so } i^{27} = i^3 = -i. 50 = 4 \cdot 12 + 2, \text{ so } i^{50} = i^2 = -1.$$

$-i$ and -1

Exercise 3 - Add complex numbers

$$\text{Real parts: } 4+2=6. \text{ Imaginary parts: } 3-5=-2.$$

$6-2i$

Exercise 4 - Subtract complex numbers

$$7-3=4. -2i-(-6i)=4i.$$

$4+4i$

Exercise 5 - Multiply

$$8-2i+12i-3i^2=8+10i+3.$$

$$11+10i$$

Exercise 6 - Square a complex number

$$9+24i+16i^2=9+24i-16.$$

$$-7+24i$$

Exercise 7 - Conjugate product

The conjugate is $5+12i$. Product: $5^2+12^2=25+144$.

$$5+12i; 169$$

Exercise 8 - Divide

Multiply by $\frac{1+i}{1+i}$: numerator $3+3i+2i+2i^2=1+5i$; denominator $1+1=2$.

$$\frac{1}{2}+\frac{5}{2}i$$

Exercise 9 - Divide a real number

$$\frac{10(3+i)}{(3-i)(3+i)}=\frac{30+10i}{10}.$$

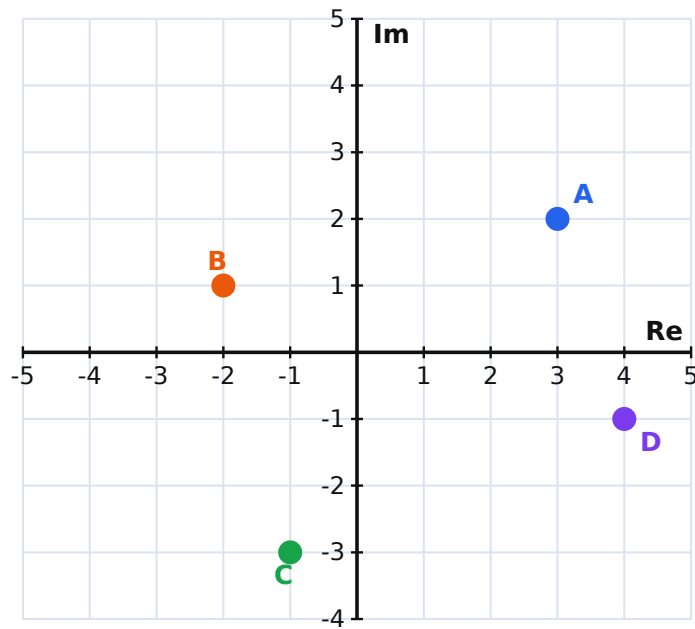
$$3+i$$

Exercise 10 - Modulus

$$\sqrt{25+144}=13. \sqrt{9+16}=5.$$

$$13 \text{ and } 5$$

Exercise 11 - Read the complex plane



$$A = 3 + 2i, B = -2 + i, C = -1 - 3i, D = 4 - i.$$

Read the real part from the horizontal axis and the imaginary part from the vertical axis.

$$|A| = \sqrt{9+4} = \sqrt{13}.$$

$$A = 3 + 2i, B = -2 + i, C = -1 - 3i, D = 4 - i; |A| = \sqrt{13}$$

Exercise 12 - A pure imaginary solution

$$x^2 = -4, \text{ so } x = \pm\sqrt{-4}.$$

$$x = \pm 2i$$

Exercise 13 - Complex roots by the formula

$$x = \frac{6 \pm \sqrt{36 - 52}}{2} = \frac{6 \pm 4i}{2}. \text{ The discriminant is negative, so the parabola never reaches the axis.}$$

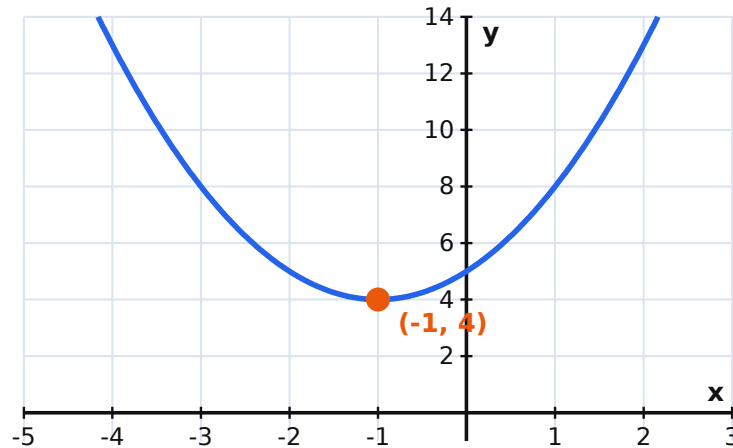
$$x = 3 \pm 2i$$

Exercise 14 - Complex roots with a leading coefficient

$$x = \frac{-2 \pm \sqrt{4 - 40}}{4} = \frac{-2 \pm 6i}{4}.$$

$$x = -\frac{1}{2} \pm \frac{3}{2}i$$

Exercise 15 - Discriminant



Negative discriminant: no x-intercepts.

(a) $16-16=0$: one real double root. (b) $1-12=-11<0$: two complex roots. (c) $25-8=17>0$: two real roots.

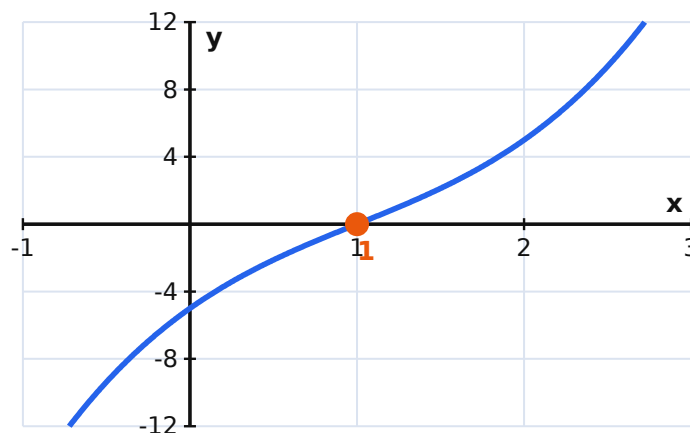
(a) one real; (b) two complex; (c) two real

Exercise 16 - Conjugate pair

The conjugate $2-3i$ is also a zero. $(x-2-3i)(x-2+3i)=(x-2)^2+9=x^2-4x+13$.

$2-3i$; $x^2-4x+13$

Exercise 17 - All zeros of a cubic



Only one real zero.

Synthetic division by $x-1$ gives x^2-2x+5 . Then $x = \frac{2 \pm \sqrt{4-20}}{2} = 1 \pm 2i$.

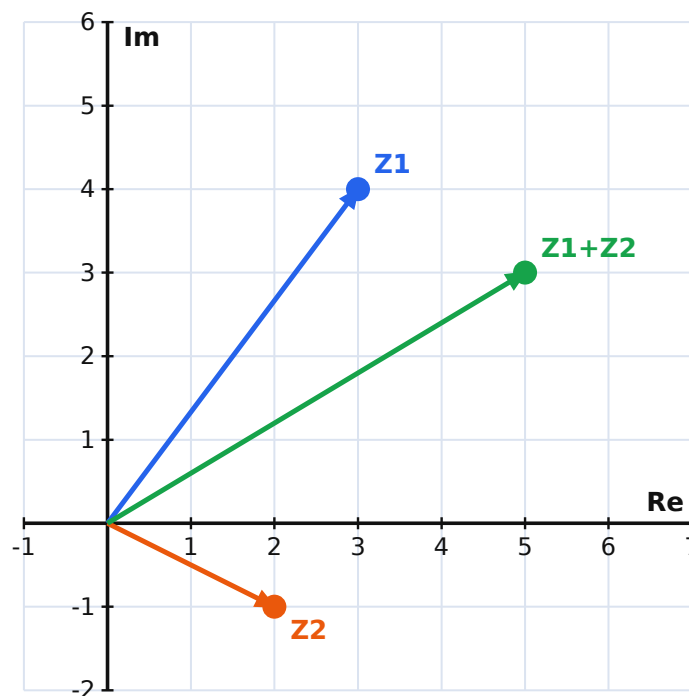
1, $1+2i$, $1-2i$

Exercise 18 - Missing zeros

Non-real zeros come in conjugate pairs: $1-i$ and $3i$.

$1-i$ and $3i$

Exercise 19 - Impedance



$$Z_1 + Z_2 = 5 + 3i.$$

$Z_1 + Z_2 = 5 + 3i$, and $\sqrt{25+9} = \sqrt{34} \approx 5.83$. $V = (2+i)(3+4i) = 6+8i+3i+4i^2 = 2+11i$.

$5+3i$ ohms; about 5.83 ohms; $V = 2+11i$ volts

Exercise 20 - Verify a root

$(1+2i)^2 = 1+4i-4 = -3+4i$. Then $-3+4i-2(1+2i)+5 = -3+4i-2-4i+5 = 0$.

The result is 0, so it is a root

Exercise 21 - Power of a complex number

$$(1+i)^2 = 1+2i+i^2 = 2i. \text{ So } (1+i)^4 = (2i)^2 = 4i^2 = -4.$$

$$-4$$