

Precalculus - Complex Numbers and Complex Zeros - Test Answer Key

Common Core N-CN.A.1, N-CN.A.2, N-CN.A.3, N-CN.C.7, N-CN.C.8, N-CN.C.9 · maths-pdf.fr/en/

Worked solutions and scoring notes for the 9-question complex numbers test. Use the scoring line under each solution to award partial credit.

Precalculus · Common Core State Standards: **N-CN.A.1, N-CN.A.2, N-CN.A.3, N-CN.C.7, N-CN.C.8, N-CN.C.9**

Question 1 - Powers and radicals [2 pts]

$$43 = 4 \cdot 10 + 3, \text{ so } i^{43} = -i. \sqrt{-48} = i\sqrt{16 \cdot 3} = 4i\sqrt{3}.$$

$$-i \text{ and } 4i\sqrt{3}$$

Scoring: 1 point for each.

Question 2 - Multiply [2 pts]

$$3 + 12i - 2i - 8i^2 = 3 + 10i + 8.$$

$$11 + 10i$$

Scoring: 1 point for the expansion, 1 point for replacing i squared.

Question 3 - Divide [2 pts]

Multiply by $\frac{2+3i}{2+3i}$: numerator $10+15i+2i+3i^2=7+17i$; denominator $4+9=13$.

$$\frac{7}{13} + \frac{17}{13}i$$

Scoring: 1 point for the conjugate step, 1 point for the result.

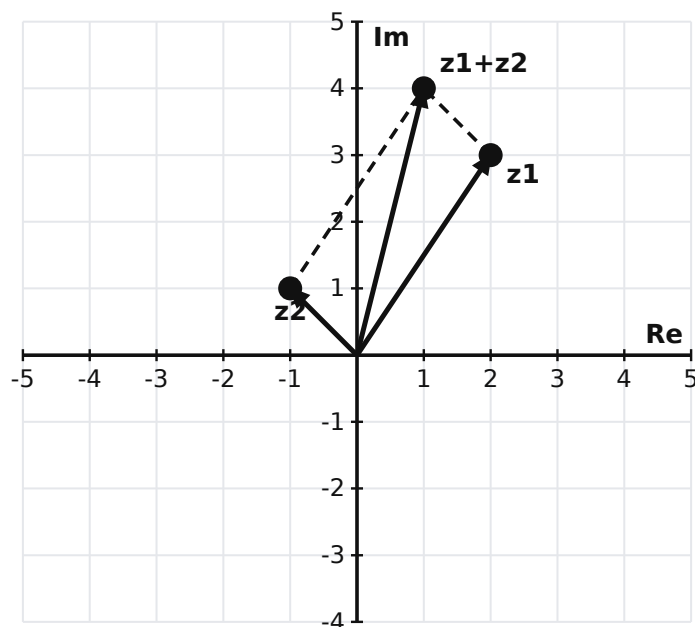
Question 4 - Modulus and conjugate [2 pts]

$$|z| = \sqrt{25+144} = 13. z\bar{z} = 25+144 = 169.$$

$$13 \text{ and } 169$$

Scoring: 1 point for each.

Question 5 - Complex plane [3 pts]



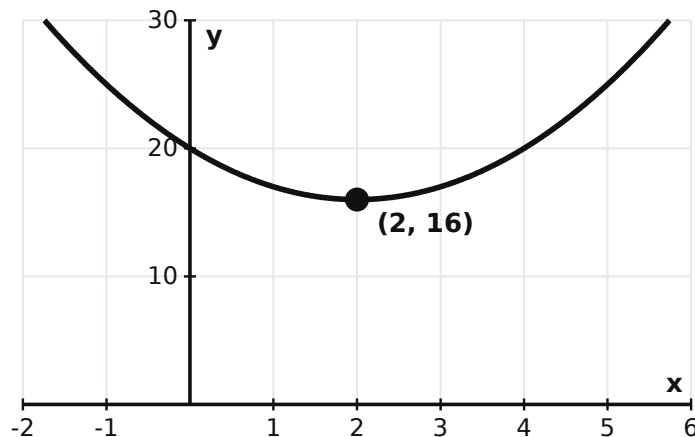
The sum is the fourth corner of the parallelogram.

(a) $z_1 = 2 + 3i$, $z_2 = -1 + i$. (b) $1 + 4i$. (c) $\sqrt{4+9} = \sqrt{13}$.

$2 + 3i$, $-1 + i$; $1 + 4i$; $\sqrt{13}$

Scoring: 1 point for each part.

Question 6 - Quadratic with complex roots [2 pts]



Lowest point (2, 16).

$x = \frac{4 \pm \sqrt{16-80}}{2} = \frac{4 \pm 8i}{2}$. The graph confirms there are no real zeros.

$$x = 2 \pm 4i$$

Scoring: 1 point for the discriminant, 1 point for the roots.

Question 7 - Classify the roots [2 pts]

$36-40 = -4 < 0$, two non-real roots. $x = \frac{-6 \pm 2i}{2}$.

$$-4; x = -3 \pm i$$

Scoring: 1 point for the discriminant and classification, 1 point for the roots.

Question 8 - Conjugate pairs and degree [3 pts]

(a) $3+i$. (b) $(x-3+i)(x-3-i) = (x-3)^2 + 1 = x^2 - 6x + 10$. (c) $(x-1)(x-2) = x^2 - 3x + 2$; times $x^2 - 6x + 10$ gives $x^4 - 9x^3 + 30x^2 - 42x + 20$.

$$3+i; x^2-6x+10; x^4-9x^3+30x^2-42x+20$$

Scoring: 1 point for each part.

Question 9 - Verify a root [2 pts]

$$(2-i)^2 = 4 - 4i + i^2 = 3 - 4i. \text{ Then } 3 - 4i - 4(2-i) + 5 = 3 - 4i - 8 + 4i + 5 = 0.$$

The value is 0

Scoring: 1 point for squaring correctly, 1 point for showing the sum is 0.

Total
20 points