

Precalculus - Conic Sections - Lesson

Common Core G-GPE.A.1, G-GPE.A.2, G-GPE.A.3 · maths-pdf.fr/en/

Conic sections are the four curves cut from a double cone: circle, parabola, ellipse and hyperbola. A circle keeps a fixed distance from its center. A parabola keeps equal distances to a focus and a directrix. An ellipse keeps a constant sum of distances to two foci, and a hyperbola a constant difference. Complete the square to reach standard form.

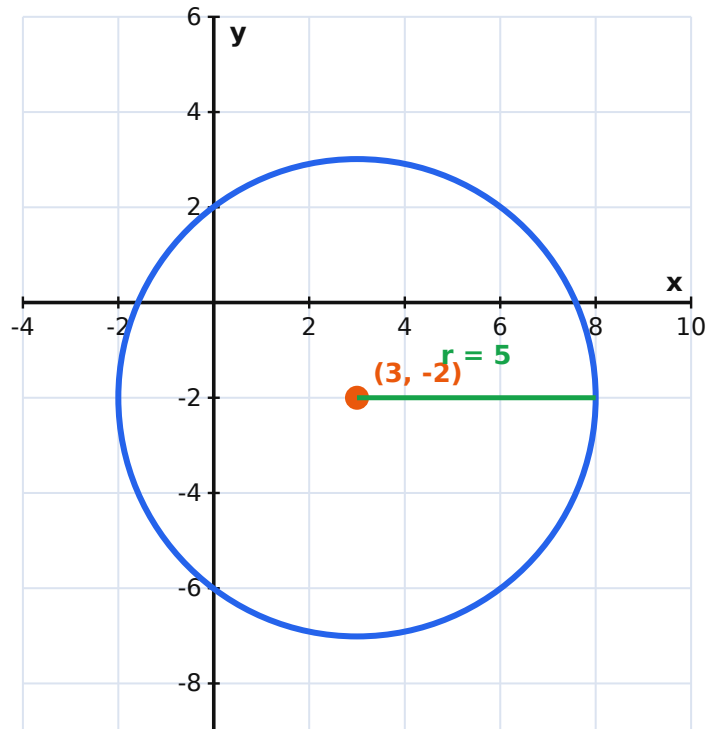
Precalculus · Common Core State Standards: **G-GPE.A.1, G-GPE.A.2, G-GPE.A.3**

Key facts

- Circle: $(x - h)^2 + (y - k)^2 = r^2$.
- Parabola: $(x - h)^2 = 4p(y - k)$; focus is p from the vertex and the directrix is on the opposite side.
- Ellipse: $x^2/a^2 + y^2/b^2 = 1$ with $c^2 = a^2 - b^2$; the foci lie on the major axis.
- Hyperbola: $x^2/a^2 - y^2/b^2 = 1$ with $c^2 = a^2 + b^2$ and asymptotes $y = \text{plus or minus } (b/a) x$.
- Complete the square to find the center or vertex.

What are conic sections?

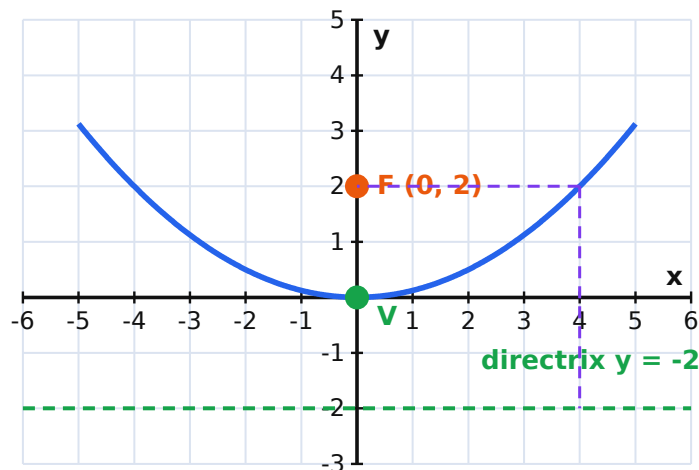
A conic section is what you get when a plane cuts through a pair of cones joined tip to tip. Depending on the angle of the cut, the outline is a circle, a parabola, an ellipse or a hyperbola, and every one of them is described by a quadratic equation in x and y . The simplest is the circle, the collection of all points lying exactly r units from a fixed center (h, k) , which satisfies $(x-h)^2 + (y-k)^2 = r^2$.



The circle $(x-3)^2+(y+2)^2=25$ has center $(3,-2)$ and radius 5.

What is a parabola?

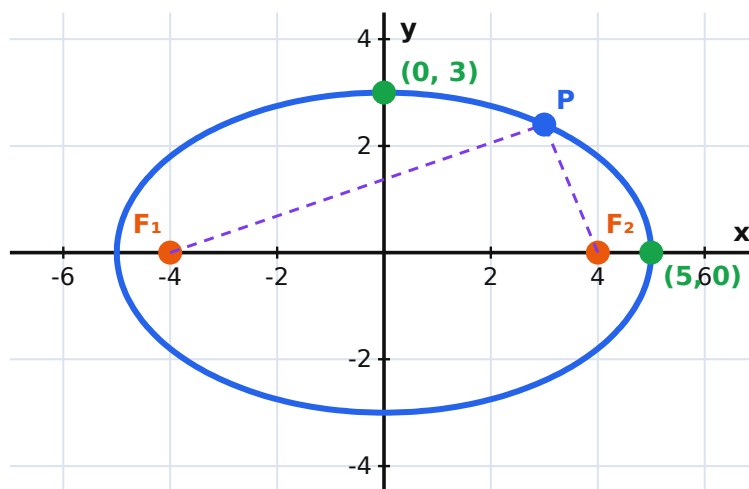
Pick a point called the **focus** and a line called the **directrix**. The parabola is made of every point whose two distances, to the focus and to the directrix, are identical. Placing the vertex at the origin with the opening upward gives $x^2=4py$: the focus sits at $(0,p)$ and the directrix is the line $y=-p$. Moving the vertex to (h,k) changes the equation to $(x-h)^2=4p(y-k)$. A negative p flips the opening, and the sideways versions are $y^2=4px$.



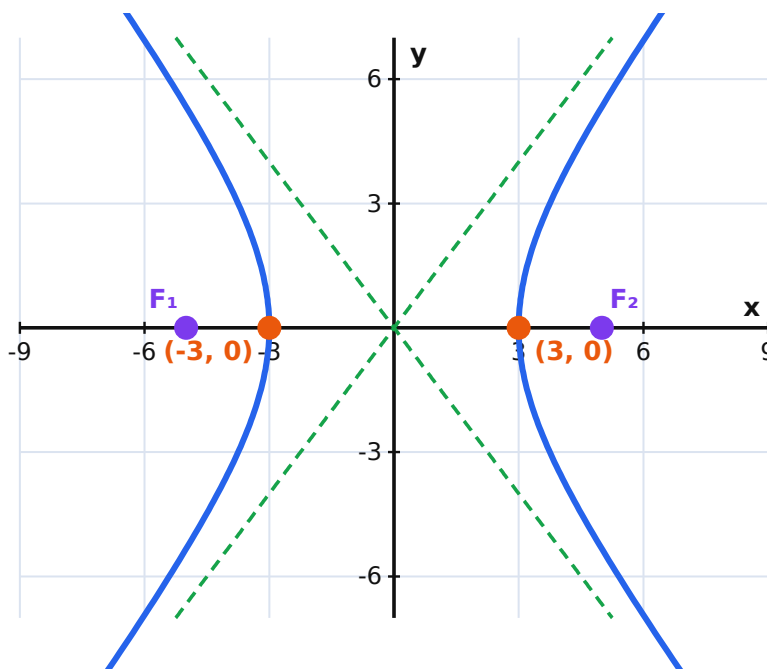
For $x^2=8y$, $4p=8$, so $p=2$: focus $(0,2)$, directrix $y=-2$.

What are an ellipse and a hyperbola?

Fix two points, the foci. An **ellipse** collects the points where the two distances to the foci always total $2a$; a **hyperbola** collects the points where those distances always differ by $2a$. For an ellipse centered at the origin and stretched along the x -axis, the equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b$, and the foci lie at $(\pm c, 0)$ with $c^2 = a^2 - b^2$. The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has $c^2 = a^2 + b^2$ and two slanted guide lines, the asymptotes $y = \pm \frac{b}{a}x$.



For $\frac{x^2}{25} + \frac{y^2}{9} = 1$: $a = 5$, $b = 3$, $c = 4$. Every point P has $PF_1 + PF_2 = 10$.



For $\frac{x^2}{9} - \frac{y^2}{16} = 1$: vertices $(\pm 3, 0)$, foci $(\pm 5, 0)$ and asymptotes $y = \pm \frac{4}{3}x$.

Conic	Standard form	Key relation
Circle	$(x-h)^2 + (y-k)^2 = r^2$	radius r

Conic	Standard form	Key relation
Parabola	$(x-h)^2 = 4p(y-k)$	focus at distance $ p $ from the vertex
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$c^2 = a^2 - b^2$
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$c^2 = a^2 + b^2$

Real equations often arrive in expanded form, so complete the square in both variables first. Then read the shape from the signs and sizes of the squared terms: matching positive coefficients mean a circle, two different positive coefficients mean an ellipse, one positive and one negative mean a hyperbola, and a single squared variable means a parabola. Engineers rely on these shapes for dishes that gather signals, rooms where whispers travel, planetary orbits and arches.

Questions about conic sections

What is the focus of a parabola?

It is the fixed point used to define the parabola. Every point on the parabola is the same distance from the focus as from the directrix. Signals reflect to the focus, which is why dishes are parabolic.

How do you find the foci of an ellipse?

Compute c from $c^2 = a^2 - b^2$, where a is the semi-major axis. The foci are c units from the center along the major axis.

How do ellipses differ from hyperbolas in the equation?

An ellipse has a plus sign between the squared terms and a hyperbola has a minus sign. For hyperbolas, $c^2 = a^2 + b^2$, and for ellipses, $c^2 = a^2 - b^2$.

What are asymptotes of a hyperbola?

They are the lines the branches approach for large $|x|$. For $x^2/a^2 - y^2/b^2 = 1$ they are $y = \pm (b/a)x$.

How do you identify a conic from a general equation?

Complete the square, then compare the form: equal positive coefficients give a circle, unequal positive coefficients give an ellipse, opposite signs give a hyperbola, and one squared variable gives a parabola.