

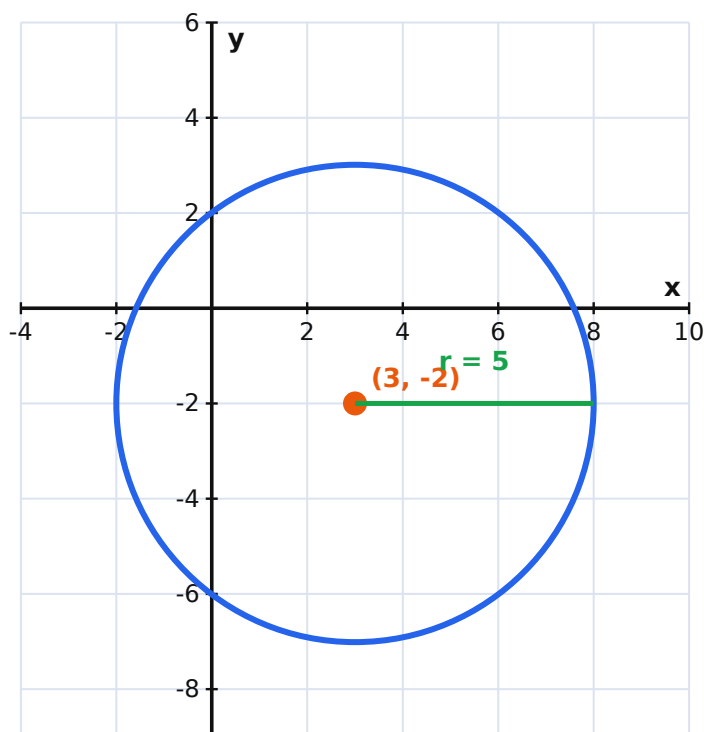
# Precalculus - Conic Sections - Answer Key

Common Core G-GPE.A.1, G-GPE.A.2, G-GPE.A.3 · [maths-pdf.fr/en/](https://maths-pdf.fr/en/)

Use this key to check each conic. Solutions show completing the square or the relation among a, b and c, with final answers in bold.

**Precalculus** · Common Core State Standards: **G-GPE.A.1, G-GPE.A.2, G-GPE.A.3**

## Exercise 1 - Equation of a circle



Center (3, -2) and radius 5.

$$(x-3)^2 + (y-(-2))^2 = 5^2.$$

$$(x-3)^2 + (y+2)^2 = 25$$

## Exercise 2 - Center and radius

Complete the squares:  $(x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 + 9 + 4$ , so  $(x-3)^2 + (y+2)^2 = 25$ .

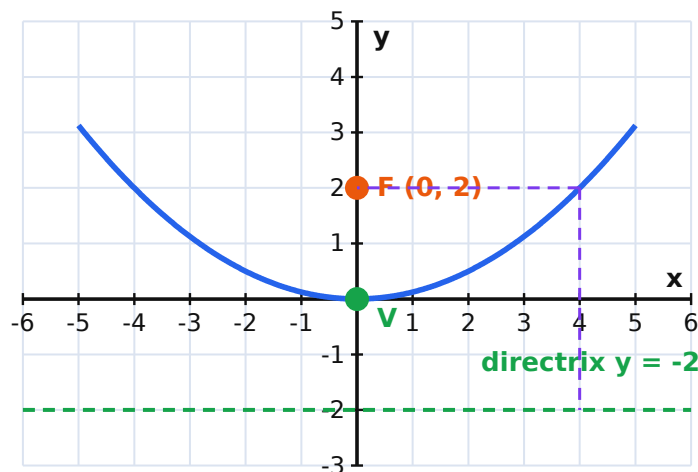
**Center** (3, -2); **radius** 5

### Exercise 3 - Circle through a point

$$r^2 = (1 - (-2))^2 + (2 - 6)^2 = 9 + 16 = 25.$$

$$(x+2)^2 + (y-6)^2 = 25$$

### Exercise 4 - Parts of a parabola



Focus (0, 2), directrix  $y = -2$ .

$4p = 8$ , so  $p = 2$ . The vertex is (0,0), the focus is (0,2), the directrix is  $y = -2$ .

**Vertex (0,0); focus (0,2); directrix  $y = -2$**

### Exercise 5 - Parabola opening left

$4p = -12$ , so  $p = -3$ . The parabola opens left: focus  $(-3,0)$ , directrix  $x = 3$ .

**Focus  $(-3,0)$ ; directrix  $x = 3$**

### Exercise 6 - Equation from focus and directrix

The vertex is the midpoint (0,0), and the parabola opens down with  $p = -5$ .  $x^2 = 4py = -20y$ .

$$x^2 = -20y$$

### Exercise 7 - Shifted parabola

$h=2$ ,  $k=-1$ ,  $4p=4$ , so  $p=1$ . The focus is 1 above the vertex, and the directrix is 1 below.

**Vertex**  $(2,-1)$ ; **focus**  $(2,0)$ ; **directrix**  $y=-2$

### Exercise 8 - Complete the square for a parabola

$y-7+4=x^2-4x+4$ , so  $(x-2)^2=y-3$ .  $4p=1$ ,  $p=\frac{1}{4}$ . The vertex is  $(2,3)$ .

$(x-2)^2=y-3$ ; **focus**  $(2, \frac{13}{4})$

### Exercise 9 - Parts of an ellipse

$a=10$ ,  $b=6$ ,  $c=\sqrt{100-36}=8$ . The major axis is horizontal.

$a=10, b=6, c=8$ ; **vertices**  $(\pm 10, 0)$ ; **foci**  $(\pm 8, 0)$

### Exercise 10 - Vertical ellipse

The larger denominator is under  $y^2$ , so the major axis is vertical:  $a^2=36$ ,  $b^2=20$ ,  $c=\sqrt{36-20}=4$ .

$(0, \pm 4)$

### Exercise 11 - Ellipse from vertices and foci

$a=6$ ,  $c=4$ , so  $b^2=a^2-c^2=36-16=20$ .

$$\frac{x^2}{36} + \frac{y^2}{20} = 1$$

### Exercise 12 - Shifted ellipse

Center  $(1, -2)$ .  $a=3$ ,  $b=2$ ,  $c=\sqrt{9-4}=\sqrt{5}$ . The major axis is horizontal.

**Center**  $(1, -2)$ ; **foci**  $(1 \pm \sqrt{5}, -2)$

### Exercise 13 - Standard form of an ellipse

Divide by 36.

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

### Exercise 14 - Parts of a hyperbola

$a = 7$ ,  $b = 24$ ,  $c = \sqrt{49 + 576} = 25$ . The asymptotes are  $y = \pm \frac{b}{a}x$ .

**Vertices**  $(\pm 7, 0)$ ; **foci**  $(\pm 25, 0)$ ;  $y = \pm \frac{24}{7}x$

### Exercise 15 - Vertical hyperbola

$a^2 = 25$ ,  $b^2 = 144$ ,  $c = \sqrt{169} = 13$ . With the  $y^2$  term first, the branches open up and down and the asymptotes are  $y = \pm \frac{5}{12}x$ .

**Foci**  $(0, \pm 13)$ ;  $y = \pm \frac{5}{12}x$

### Exercise 16 - Hyperbola from vertices and foci

$a = 4$ ,  $c = 5$ , so  $b^2 = c^2 - a^2 = 9$ .

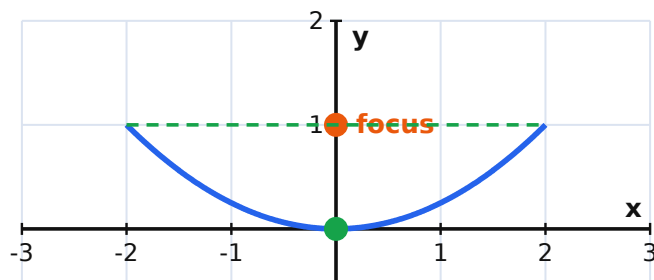
$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

### Exercise 17 - Identify the conic

(a) equal coefficients, same sign: circle. (b) one squared variable: parabola. (c) opposite signs: hyperbola. (d) same sign, different coefficients: ellipse.

**(a) circle; (b) parabola; (c) hyperbola; (d) ellipse**

### Exercise 18 - Satellite dish

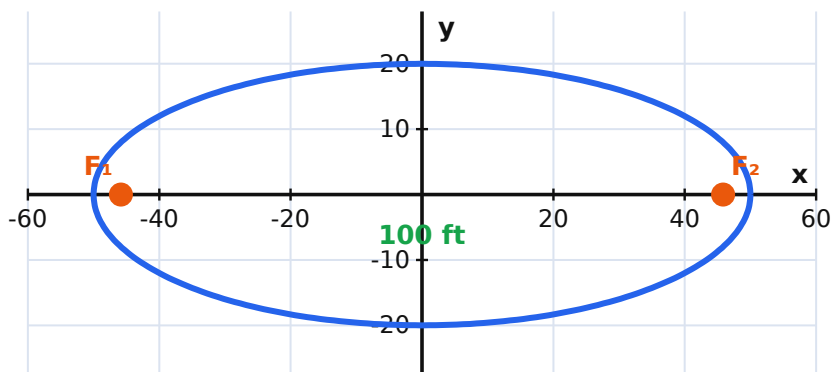


The focus is 1 foot above the vertex.

The rim point is (2,1):  $4 = 4p(1)$ , so  $p = 1$ .

$x^2 = 4y$ ; **focus 1 ft above the vertex**

### Exercise 19 - Whispering gallery

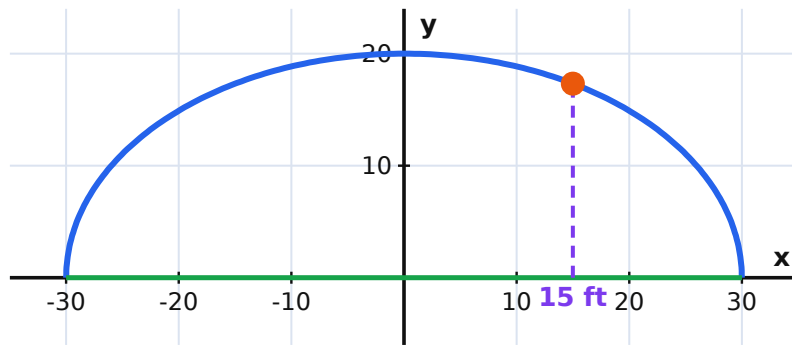


Foci about 45.8 feet from the center.

$a = 50$ ,  $b = 20$ ,  $c = \sqrt{2500 - 400} \approx 45.83$ . The foci are  $2c$  apart.

$2c \approx 91.7$  ft

### Exercise 20 - Elliptical arch



Height 17.32 feet at 15 feet from the center.

$a = 30$ ,  $b = 20$ :  $\frac{x^2}{900} + \frac{y^2}{400} = 1$ . At  $x = 15$ :  $y^2 = 400\left(1 - \frac{225}{900}\right) = 300$ , so  $y = \sqrt{300} \approx 17.32$ .

$\frac{x^2}{900} + \frac{y^2}{400} = 1$ ; 17.32 ft