

Precalculus - Exponential and Logarithmic Equations - Lesson

Common Core F-LE.A.4, F-BF.B.5, A-REI.D.11 · maths-pdf.fr/en/

To solve an exponential equation, isolate the power, then either match the bases or take a logarithm of both sides and use the power rule to bring the exponent down. To solve a logarithmic equation, combine the logs, rewrite in exponential form and solve. Always check logarithmic solutions, because an argument that is zero or negative makes a solution extraneous.

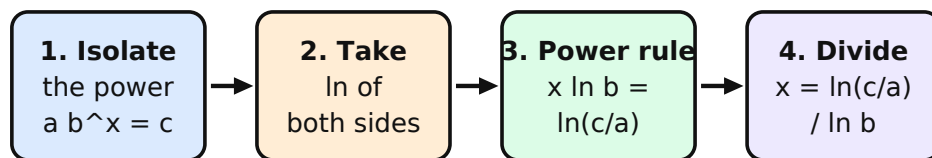
Precalculus · Common Core State Standards: **F-LE.A.4, F-BF.B.5, A-REI.D.11**

Key facts

- Same base: if $b^m = b^n$ then $m = n$.
- Different bases: take $\ln$ of both sides, then $x \ln b = \ln c$.
- $\log_b u = c$ is equivalent to $u = b^c$.
- Check log solutions: arguments must be positive; reject extraneous values.
- Continuous doubling time is $\ln 2$ divided by the rate r .

How do you solve an exponential equation?

If both sides can be written with the same base, set the exponents equal: $9^x = 27$ becomes $3^{2x} = 3^3$, so $2x = 3$ and $x = \frac{3}{2}$. When the bases cannot be matched, take a logarithm of both sides. The power rule brings the exponent down: $3^x = 20$ gives $x \ln 3 = \ln 20$, so $x = \frac{\ln 20}{\ln 3} \approx 2.727$. Always isolate the exponential part first.



Four steps for an equation of the form $a b^x = c$.

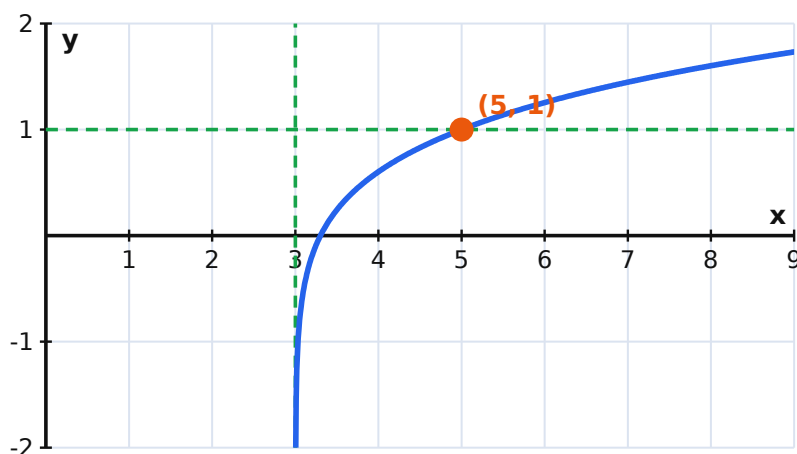
For equations with base e , use the natural log, since $\ln e^u = u$. For example, $e^{2x} = 15$ gives $2x = \ln 15$, so $x = \frac{\ln 15}{2} \approx 1.354$.

How do you solve a logarithmic equation?

1. Combine all logarithms into one using the product, quotient and power rules.

2. Rewrite in exponential form: $\log_b u = c$ means $u = b^c$.
3. Solve the resulting equation.
4. Check each answer in the original equation, because arguments must be positive.

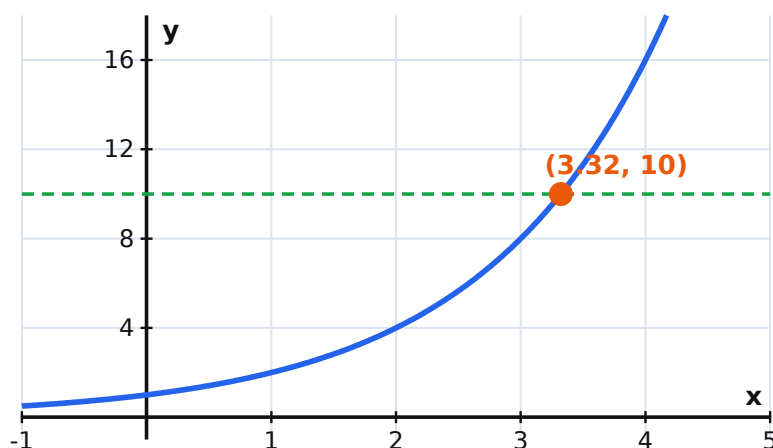
Example: Solve $\log x + \log(x-3) = 1$. Combine: $\log(x(x-3)) = 1$, so $x^2 - 3x = 10$. Then $x^2 - 3x - 10 = (x-5)(x+2) = 0$. The value $x = -2$ makes $\log(-2)$ undefined, so it is extraneous. The solution is $x = 5$.



The curve $y = \log x + \log(x-3)$ exists only for $x > 3$ and meets $y = 1$ once, at $x = 5$.

Why can you solve an equation by graphing?

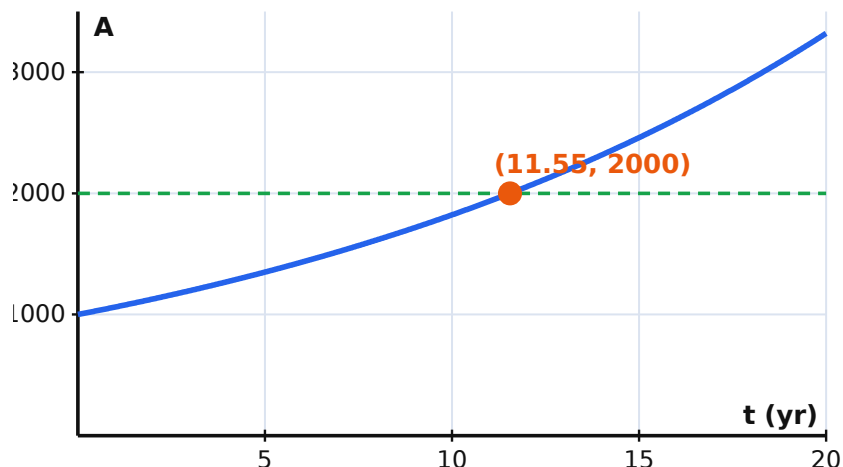
Every solution of $f(x) = g(x)$ is the x -coordinate of an intersection point of $y = f(x)$ and $y = g(x)$. Graphing is a good check and works even when algebra is hard. The graph of $y = 2^x$ crosses $y = 10$ at $x = \log_2 10 \approx 3.32$.



The solution of $2^x = 10$ is the x -value where the curve meets the line.

How are these equations used in applications?

Doubling time, half-life and time to reach a savings goal all require solving for an exponent. With continuous growth $A = Pe^{rt}$, the doubling time satisfies $e^{rt} = 2$, so $t = \frac{\ln 2}{r}$. At 6% that is $\frac{0.6931}{0.06} \approx 11.55$ years, regardless of the starting amount.



Money growing continuously at 6 percent doubles in about 11.55 years.

When you report an answer from a real situation, round sensibly and include units, and check that it is reasonable: a result of 11.55 years for doubling at 6% matches the rule of thumb that 72 divided by the percent rate estimates the doubling time.

Questions about exponential and logarithmic equations

When can I just set exponents equal?

When both sides can be written as a power of the same base, such as $2^x = 32 = 2^5$. Then $x = 5$.

Which logarithm should I use to solve $3^x = 20$?

Any base works, as long as you use the same one on both sides. Natural log or common log are the easiest to compute: $x = \ln 20 / \ln 3 = \log 20 / \log 3$.

What is an extraneous solution in a log equation?

It is an answer that satisfies the algebra but makes the argument of a logarithm zero or negative in the original equation, so it must be rejected.

How do I find a doubling time?

Set the growth model equal to twice the starting amount and solve for t. For continuous

growth at rate r , $t = \ln 2 / r$.

Can I solve these equations by graphing?

Yes. Graph each side as a function and find the x-coordinate of the intersection. It is a good way to check an algebraic answer.