

Precalculus - Exponential and Logarithmic Equations - Answer Key

Common Core F-LE.A.4, F-BF.B.5, A-REI.D.11 · maths-pdf.fr/en/

Use this key to check each equation. Solutions show isolating the exponential, taking logs, converting forms and checking arguments, with final answers in bold.

Precalculus · Common Core State Standards: **F-LE.A.4, F-BF.B.5, A-REI.D.11**

Exercise 1 - Same base

$$64 = 2^6.$$

$$x = 6$$

Exercise 2 - Same base with a shift

$$125 = 5^3, \text{ so } x-1 = 3.$$

$$x = 4$$

Exercise 3 - Rewrite the base

$$9^x = 3^{2x} \text{ and } 27 = 3^3, \text{ so } 2x = 3.$$

$$x = \frac{3}{2}$$

Exercise 4 - Take a log

$$x \ln 3 = \ln 20, \text{ so } x = \frac{\ln 20}{\ln 3} \approx 2.727.$$

$$x \approx 2.727$$

Exercise 5 - Isolate first

$$\text{Divide by 4: } 3^x = 25. \text{ Then } x = \frac{\ln 25}{\ln 3} \approx 2.930.$$

$$x \approx 2.930$$

Exercise 6 - Base e

$$2x = \ln 15, \text{ so } x = \frac{\ln 15}{2} \approx 1.354.$$

$$x \approx 1.354$$

Exercise 7 - Base e with isolation

$$2e^{3x} = 10, e^{3x} = 5, 3x = \ln 5, \text{ so } x = \frac{\ln 5}{3} \approx 0.536.$$

$$x \approx 0.536$$

Exercise 8 - Simple log equation

$$x + 3 = 2^4 = 16.$$

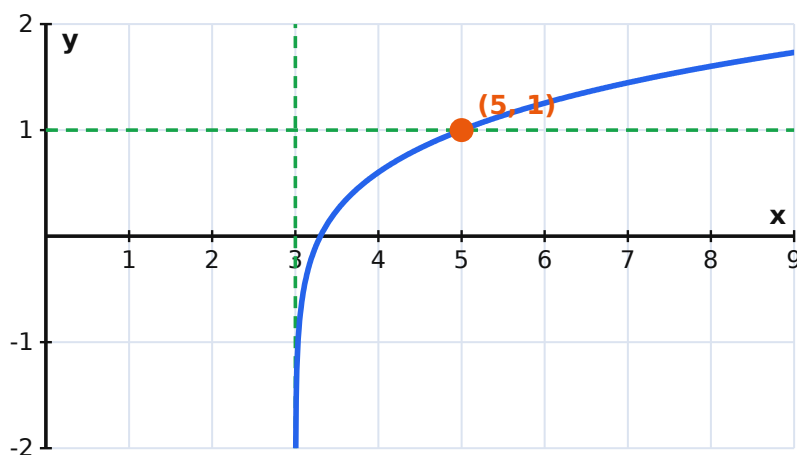
$$x = 13$$

Exercise 9 - Natural log equation

$$x - 1 = e^3 \approx 20.09, \text{ so } x = 1 + e^3 \approx 21.09.$$

$$x \approx 21.09$$

Exercise 10 - Combine logs



The graph meets $y = 1$ at $x = 5$.

$$\log(x(x-3)) = 1, \text{ so } x^2 - 3x = 10 \text{ and } (x-5)(x+2) = 0. x = -2 \text{ makes } \log(-2) \text{ undefined.}$$

$$x=5$$

Exercise 11 - Combine base-2 logs

$$x(x-2)=2^3=8, \text{ so } x^2-2x-8=0 \text{ and } (x-4)(x+2)=0. \text{ Reject } x=-2.$$

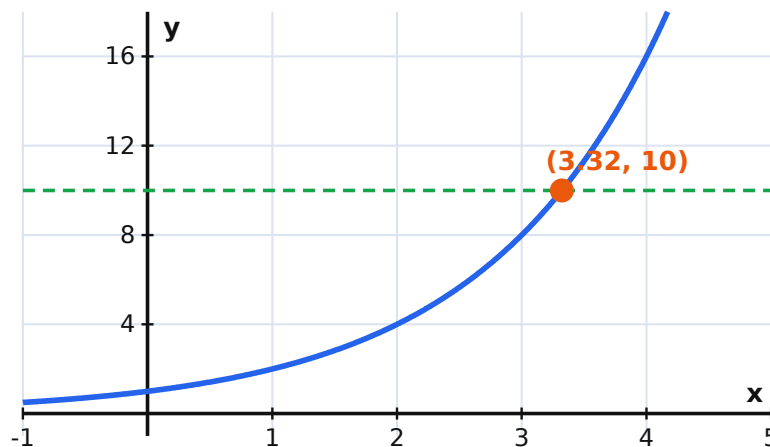
$$x=4$$

Exercise 12 - Difference of logs

$$\frac{x+6}{x-1}=2^3=8, \text{ so } x+6=8x-8 \text{ and } 7x=14, x=2. \text{ Check: both arguments are positive.}$$

$$x=2$$

Exercise 13 - Solve by graphing



Intersection at x of about 3.32.

The curve meets the line near $x=3.3$. Exactly, $x=\frac{\ln 10}{\ln 2} \approx 3.32$.

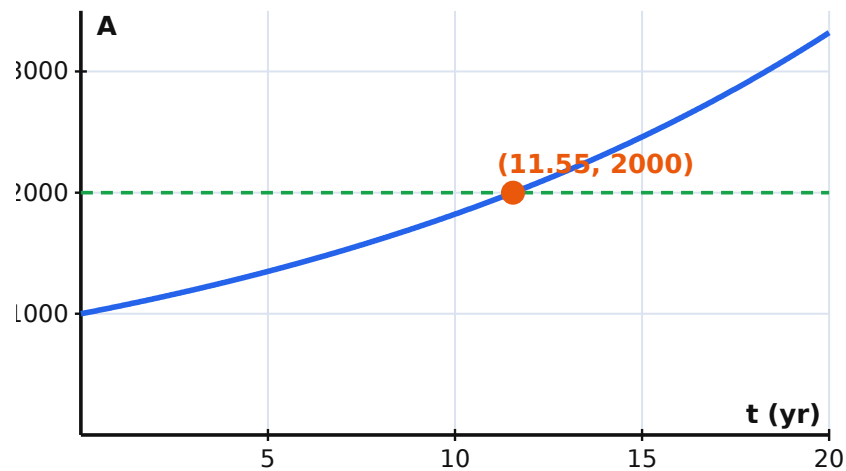
$$x \approx 3.32$$

Exercise 14 - Savings goal

$$4000(1.05)^t=6000, \text{ so } 1.05^t=1.5 \text{ and } t=\frac{\ln 1.5}{\ln 1.05} \approx 8.31.$$

$$t \approx 8.31 \text{ years}$$

Exercise 15 - Doubling time



The balance reaches 2,000 at t of about 11.55.

$$e^{0.06t} = 2, \text{ so } t = \frac{\ln 2}{0.06} \approx 11.55.$$

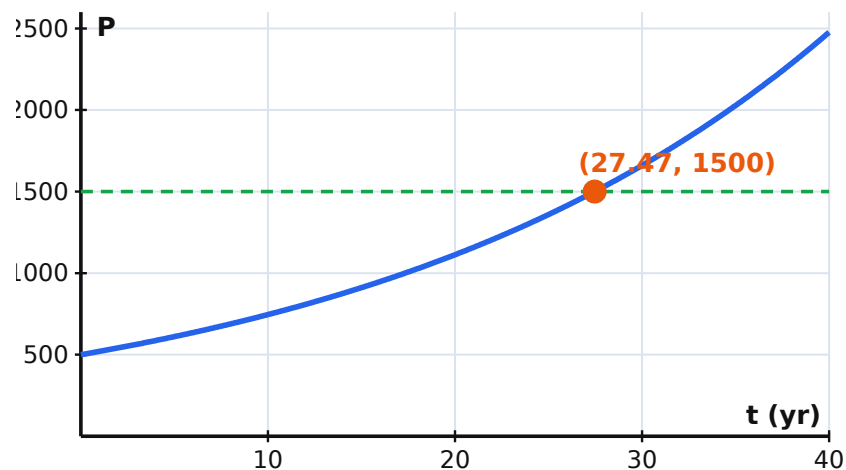
11.55 years

Exercise 16 - Half-life

$$50 \text{ g: } \left(\frac{1}{2}\right)^{t/12} = \frac{1}{4}, \text{ so } \frac{t}{12} = 2 \text{ and } t = 24. \quad 30 \text{ g: } \left(\frac{1}{2}\right)^{t/12} = 0.15, \text{ so } t = 12 \cdot \frac{\ln(200/30)}{\ln 2} \approx 32.84.$$

24 days; 32.84 days

Exercise 17 - Population growth



Reaches 1,500 at about 27.47 years.

$$e^{0.04t} = 3, \text{ so } t = \frac{\ln 3}{0.04} \approx 27.47.$$

27.47 years

Exercise 18 - Hydrogen concentration

$$[\text{H}^+] = 10^{-5.3} \approx 5.01 \times 10^{-6}.$$

$$\approx 5.01 \times 10^{-6}$$

Exercise 19 - Sound intensity

$$8.5 = \log \frac{I}{I_0}, \text{ so } \frac{I}{I_0} = 10^{8.5} \approx 3.16 \times 10^8.$$

$$\approx 3.16 \times 10^8$$

Exercise 20 - Different bases

$$(x+1)\ln 2 = x\ln 3, \text{ so } x\ln 2 + \ln 2 = x\ln 3 \text{ and } \ln 2 = x(\ln 3 - \ln 2). \text{ Then } x = \frac{\ln 2}{\ln 3 - \ln 2} = \frac{\ln 2}{\ln 1.5} \approx 1.710.$$

$$x \approx 1.710$$