

Precalculus - Exponential Functions and the Number e - Lesson

Common Core F-LE.A.2, F-IF.C.8b, F-LE.B.5, F-IF.C.7e · maths-pdf.fr/en/

An exponential function $f(x) = a b^x$ has the variable in the exponent. The number a is the starting value, and b is the factor by which the output is multiplied each time x increases by 1. If $b > 1$ the function grows; if $0 < b < 1$ it decays. Compound interest uses $A = P(1 + r/n)^{(nt)}$.

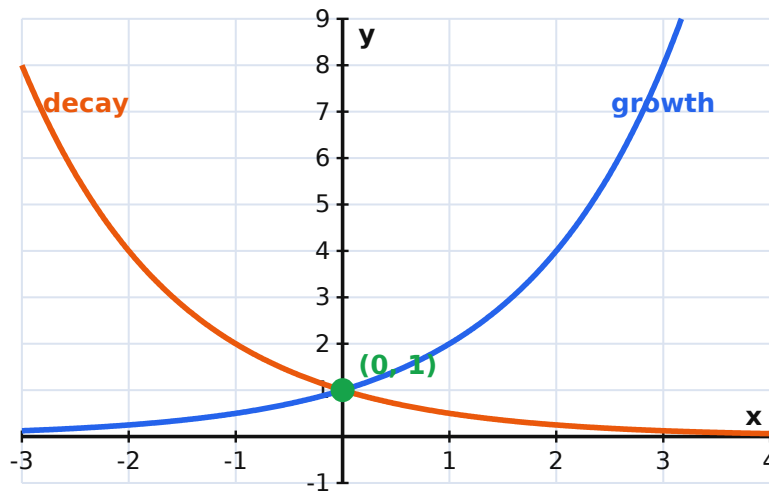
Precalculus · Common Core State Standards: **F-LE.A.2, F-IF.C.8b, F-LE.B.5, F-IF.C.7e**

Key facts

- $f(x) = a b^x$: $a = f(0)$ and b = growth or decay factor.
- A rate r per period gives $b = 1 + r$ (growth) or $b = 1 - r$ (decay).
- The parent graph has the horizontal asymptote $y = 0$; $y = a b^{(x - h)} + k$ has asymptote $y = k$.
- Compound interest: $A = P(1 + r/n)^{(nt)}$; continuous: $A = Pe^{(rt)}$.
- e is about 2.71828, the limit of $(1 + 1/n)^n$.

What is an exponential function?

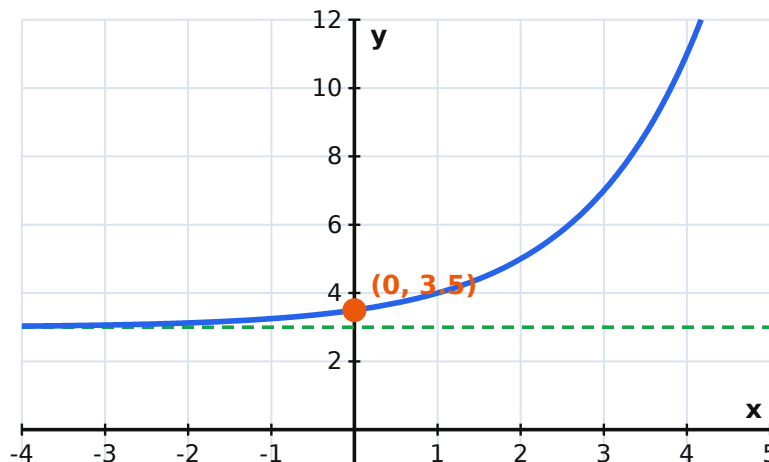
An exponential function has the form $f(x) = a b^x$ with $a \neq 0$, $b > 0$ and $b \neq 1$. The input is in the exponent. The number a is the initial value $f(0)$, and b is the growth or decay factor: each time x increases by 1, the output is multiplied by b . If $b > 1$ the function models growth; if $0 < b < 1$ it models decay. A rate of r per period gives $b = 1 + r$, so 8% growth means $b = 1.08$, and 15% decay means $b = 0.85$.



Both graphs pass through $(0,1)$ and approach the x -axis, the horizontal asymptote.

How do transformations change the graph?

The graph of $y = a b^{x-h} + k$ is the parent graph shifted right h and up k . The horizontal asymptote moves from $y=0$ to $y=k$. The domain stays all real numbers, and the range becomes $y > k$ when $a > 0$.



$y = 2^{x-1} + 3$: asymptote $y = 3$, range $(3, \infty)$.

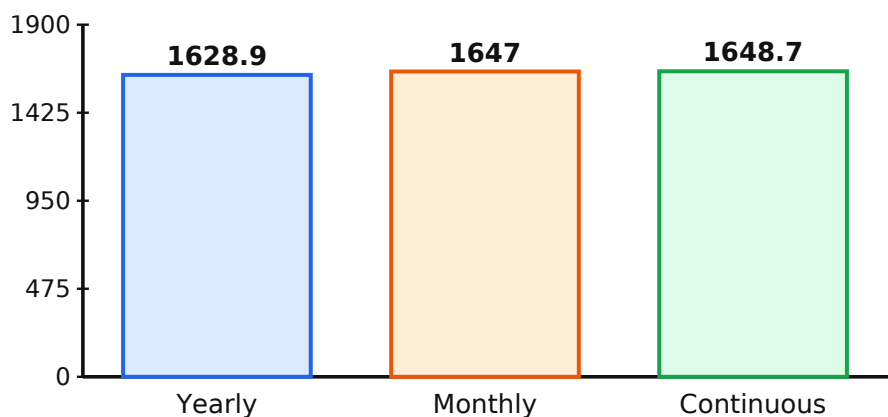
How does compound interest work?

Compound interest formulas

For principal P , annual rate r , n compounding periods per year and t years: $A = P\left(1 + \frac{r}{n}\right)^{nt}$.
For continuous compounding: $A = Pe^{rt}$.

The more often interest is added, the more you earn, but the gain levels off. For \$1,000 at 5%

for 10 years, yearly compounding gives \$1,628.89, monthly gives \$1,647.01, and continuous gives \$1,648.72.



More frequent compounding earns slightly more.

What is the number e?

The number $e \approx 2.71828$ is the value that $\left(1 + \frac{1}{n}\right)^n$ approaches as n grows without bound. It is the natural base because the function e^x has a graph whose slope at each point equals its height.

n	1	10	100	1,000	10,000
$(1 + 1/n)^n$	2.0000	2.5937	2.7048	2.7169	2.7181

The values of $\left(1 + \frac{1}{n}\right)^n$ settle toward $e \approx 2.71828$.

Exponential models appear everywhere. A population that grows by a fixed percent each year, a car that loses a fixed percent of its value, and a sample of a radioactive element that loses half its mass every few days all follow the same pattern: repeated multiplication by the same factor. A linear function adds the same amount each step, while an exponential function multiplies by the same factor, so an exponential model eventually outgrows any linear model with the same starting value.

To write a model from data, divide consecutive outputs to find b , then use $f(0)$ for a . Two points (x_1, y_1) and (x_2, y_2) with one unit between the inputs give $b = \frac{y_2}{y_1}$.

Questions about exponential functions and the number e

How do you tell growth from decay?

Look at the base. If b is greater than 1 the function grows, and if b is between 0 and 1 it decays. For $y = 200(0.85)^x$ the base 0.85 means a 15 percent decrease per step.

How do you find the equation from two points?

Divide the y-values to get b (adjusting for the distance between the x-values), then solve for a. For (1, 6) and (3, 24), b squared equals 4, so $b = 2$ and $a = 3$.

What is the difference between compound and continuous interest?

Compound interest adds interest n times a year using $(1 + r/n)^{nt}$. Continuous interest is the limit as n grows, $A = Pe^{rt}$, which always earns slightly more.

What does the horizontal asymptote mean?

The graph gets closer and closer to the asymptote but never reaches it. For $y = 2^x + 3$ the asymptote is $y = 3$, so every output is greater than 3.

Why is e important?

It is the natural base for growth that happens continuously. Models for population, radioactive decay and continuously compounded interest all use e.