

Precalculus - Exponential Functions and the Number e - Answer Key

Common Core F-LE.A.2, F-IF.C.8b, F-LE.B.5, F-IF.C.7e · maths-pdf.fr/en/

Use this key to check each exponential model. Every computation shows the substitution into the formula, and final answers are in bold.

Precalculus · Common Core State Standards: **F-LE.A.2, F-IF.C.8b, F-LE.B.5, F-IF.C.7e**

Exercise 1 - Growth or decay?

(a) $b = 1.08 > 1$: growth, $b = 1 + 0.08$. (b) $b = 0.85 < 1$: decay, $b = 1 - 0.15$.

(a) growth, 8%; (b) decay, 15%

Exercise 2 - Evaluate

$f(0) = 3$, $f(1) = 6$, $f(4) = 3(16)$.

3, 6, 48

Exercise 3 - Equation from two points

$a = f(0) = 5$. $b = \frac{15}{5} = 3$.

$y = 5 \cdot 3^x$

Exercise 4 - Equation from points two apart

Dividing: $b^2 = \frac{24}{6} = 4$, so $b = 2$. Then $a \cdot 2 = 6$, $a = 3$.

$y = 3 \cdot 2^x$

Exercise 5 - Town population

$P(t) = 4000(1.03)^t$. $P(10) = 4000(1.3439) \approx 5,376$.

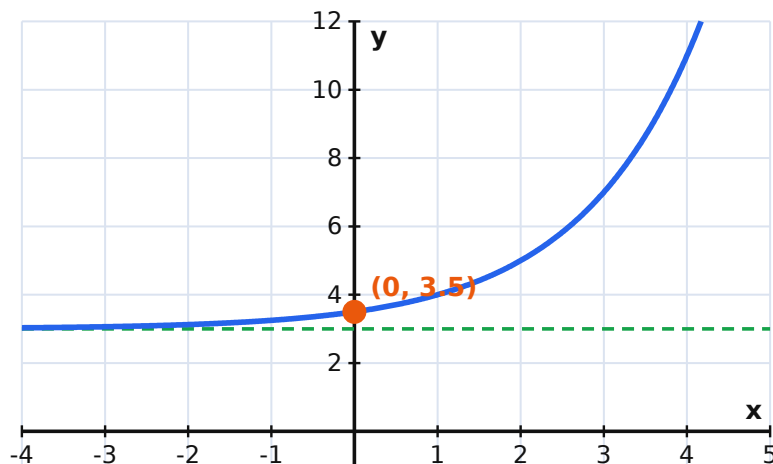
$P(t) = 4000(1.03)^t$; **about 5,376 people**

Exercise 6 - Half-life

$$A(t) = 80\left(\frac{1}{2}\right)^{t/5}. \text{ At } t = 15: 80\left(\frac{1}{2}\right)^3 = 10.$$

10 mg

Exercise 7 - Transformed exponential



Asymptote $y = 3$; y-intercept $(0, 3.5)$.

The graph is 2^x shifted right 1 and up 3. Asymptote $y = 3$. The y-intercept: $2^{-1} + 3 = 3.5$.

$y = 3$; **domain all reals; range** $(3, \infty)$; **y-int** 3.5

Exercise 8 - Equal bases

$32 = 2^5$, so the exponents must match.

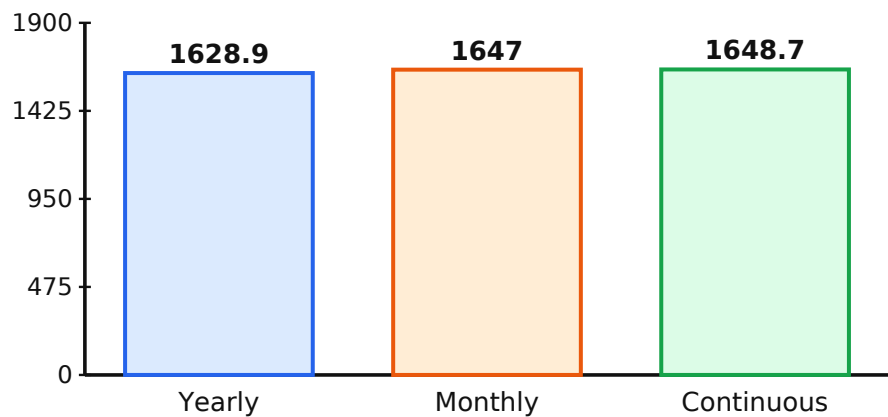
$$x = 5$$

Exercise 9 - Quarterly compounding

$$A = 2000\left(1 + \frac{0.04}{4}\right)^{4 \cdot 6} = 2000(1.01)^{24} \approx \$2,539.47.$$

\$2,539.47

Exercise 10 - Compare compounding



Yearly, monthly and continuous balances.

Yearly: $1000(1.05)^{10} \approx \$1,628.89$. Monthly: $1000\left(1 + \frac{0.05}{12}\right)^{120} \approx \$1,647.01$. Continuous: $1000e^{0.5} \approx \$1,648.72$.

\$1,628.89; \$1,647.01; \$1,648.72

Exercise 11 - Continuous growth

$$A = 500e^{0.035 \cdot 8} = 500e^{0.28} \approx \$661.56.$$

\$661.56

Exercise 12 - Approach e

n	1	10	100	1,000	10,000
$(1+1/n)^n$	2.0000	2.5937	2.7048	2.7169	2.7181

The $n = 100$ value is 2.7048.

$(1.01)^{100} \approx 2.7048$, which is about 0.0135 below e .

2.7048, close to e but still smaller

Exercise 13 - Bacteria

$$N(12) = 500 \cdot 3^3 = 13,500. \text{ For } 1500: 3^{1/4} = 3, \text{ so } \frac{1}{4} = 1.$$

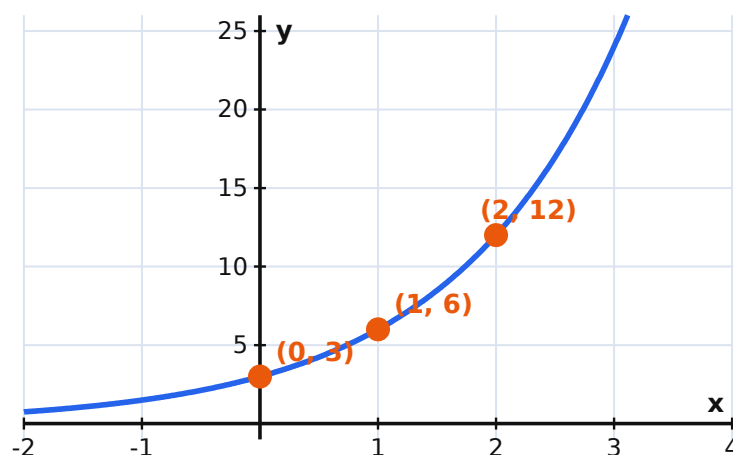
13,500 **bacteria**; $t = 4$ **hours**

Exercise 14 - Car depreciation

$$V(t) = 22000(0.88)^t. \quad V(3) = 22000(0.681472) \approx \$14,992.38.$$

$$V(t) = 22000(0.88)^t; \$14,992.38$$

Exercise 15 - Equation from a graph



Each step multiplies by 2.

$$a = 3; \frac{6}{3} = \frac{12}{6} = 2. \text{ So } f(x) = 3 \cdot 2^x \text{ and } f(5) = 3 \cdot 32 = 96.$$

$$f(x) = 3 \cdot 2^x; f(5) = 96$$

Exercise 16 - Linear versus exponential

$$f(1) = 150 < 160. \quad f(2) = 225 > 220. \quad f(3) = 337.5 > 280. \quad f(4) = 506.25 > 340.$$

f **overtakes** g **at** $x = 2$ **and stays above**

Exercise 17 - Constant percent growth

$$1200b^2 = 1728, \text{ so } b^2 = 1.44 \text{ and } b = 1.2.$$

20% per year

Exercise 18 - Find the principal

$$P(1.06)^3 = 1191.02, \text{ so } P = \frac{1191.02}{1.191016} \approx \$1,000.00.$$

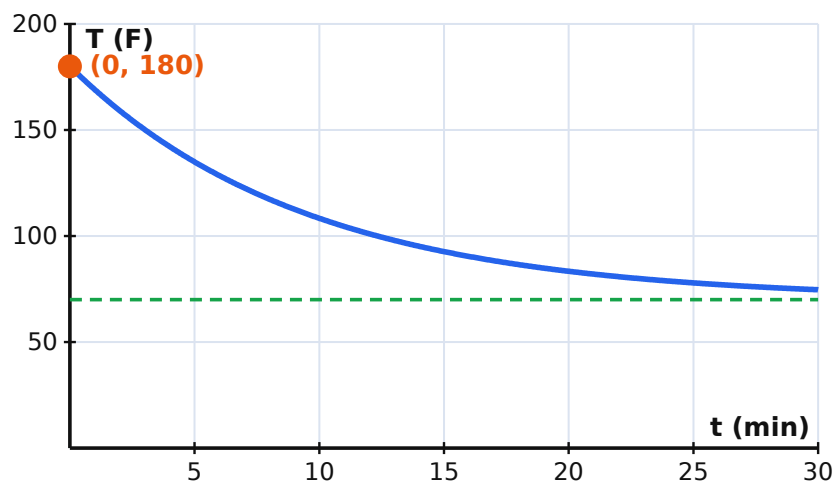
\$1,000

Exercise 19 - Doubling time

$$1.07^{10} \approx 1.967 < 2, \quad 1.07^{11} \approx 2.105 \geq 2.$$

11 years

Exercise 20 - Cooling coffee



The curve approaches 70 degrees.

$T(0) = 70 + 110 = 180$. $T(5) = 70 + 110(0.59049) \approx 135.0$. As t grows, $0.9^t \rightarrow 0$, so $T \rightarrow 70$ (the room temperature).

180°F; about 135.0°F; 70°F