

Precalculus - Combining and Composing Functions - Lesson

Common Core F-BF.A.1b, F-BF.A.1c · maths-pdf.fr/en/

Composition applies one function to the output of another: $(f \circ g)(x) = f(g(x))$ means evaluate g first, then f . Functions can also be added, subtracted, multiplied or divided at each x . The order of composition usually matters, and the domain of $f(g(x))$ needs x in the domain of g and $g(x)$ in the domain of f .

Precalculus · Common Core State Standards: **F-BF.A.1b, F-BF.A.1c**

Key facts

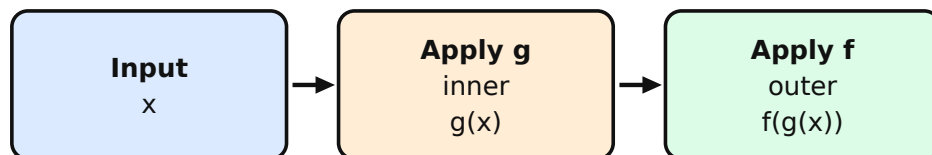
- $(f+g)(x) = f(x) + g(x)$; the same pattern holds for difference and product.
- $(f/g)(x) = f(x)/g(x)$ requires $g(x)$ not equal to 0.
- $(f \circ g)(x) = f(g(x))$: work from the inside out.
- In general $f(g(x))$ is not equal to $g(f(x))$.
- Domain of $f(g(x))$: x must be in the domain of g , and $g(x)$ in the domain of f .

How do you combine two functions?

Functions can be added, subtracted, multiplied and divided by combining their outputs at the same input. $(f+g)(x) = f(x) + g(x)$, $(f-g)(x) = f(x) - g(x)$, $(fg)(x) = f(x) \cdot g(x)$, and $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ with $g(x) \neq 0$. The domain of the result keeps only the x -values that are in both domains, and a quotient also removes every x where $g(x) = 0$.

Composition

The **composition** $(f \circ g)(x) = f(g(x))$ means: apply g first, then apply f to the result. Read it 'f of g of x.'



In $f(g(x))$, the inner function g acts first and the outer function f acts second.

How do you evaluate a composite function?

Work from the inside out. If $f(x) = 3x - 2$ and $g(x) = x^2 + 1$, then $g(2) = 5$ and $f(g(2)) = f(5) = 13$. To get a formula, substitute the whole expression $g(x)$ wherever f has an x :
 $f(g(x)) = 3(x^2 + 1) - 2 = 3x^2 + 1$.

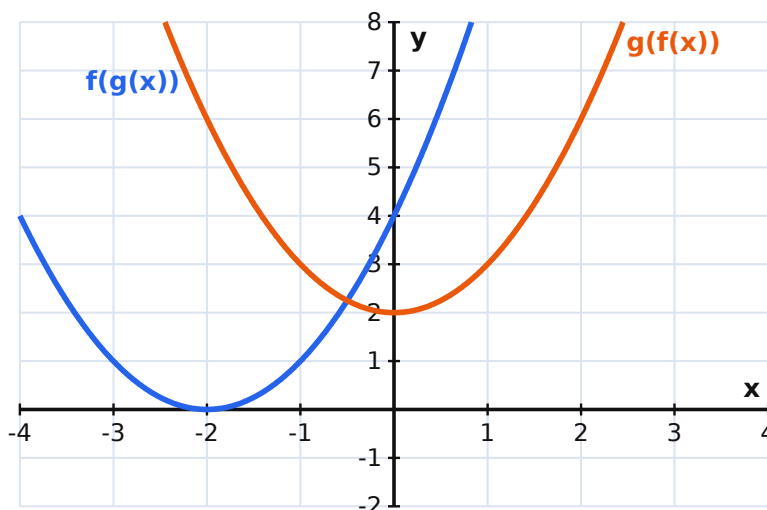
A table works the same way. In the table below, $g(1) = 0$ and $f(0) = 2$, so $f(g(1)) = 2$.

x	0	1	2	3
f(x)	2	5	1	4
g(x)	3	0	2	1

Values of f and g for $x = 0, 1, 2, 3$.

Does the order of composition matter?

Usually yes. With $f(x) = x^2$ and $g(x) = x + 2$, we get $f(g(x)) = (x + 2)^2$ but $g(f(x)) = x^2 + 2$. These are different functions, and their graphs have different vertices.



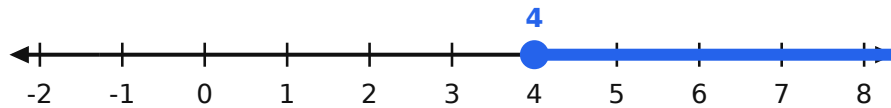
The two orders of composition give two different parabolas.

What is the domain of a composite function?

For $f(g(x))$, an input x is allowed only if (1) x is in the domain of g , and (2) $g(x)$ is in the domain of f .

1. Find the domain of the inner function g .
2. Find which outputs of g the outer function f accepts.
3. Solve for the x -values that make $g(x)$ one of those outputs.

Example: Let $f(x) = \sqrt{x}$ and $g(x) = x - 4$. We need $g(x) \geq 0$, so $x - 4 \geq 0$ and $x \geq 4$. The domain of $f \circ g$ is $[4, \infty)$. Notice that g alone accepts every real number; it is the outer function f that cuts the domain down. Always check both stages, because a composite can have a smaller domain than either function alone.



Domain of the composite: all x from 4 rightward.

You can also run the process backward. To decompose $h(x) = (5x - 1)^4$, let the inner function be $g(x) = 5x - 1$ and the outer function $f(x) = x^4$.

Questions about combining and composing functions

What does $f \circ g$ mean?

It means the composite function $f(g(x))$. Apply g to x first, then apply f to that result. The small circle is read as 'composed with.'

Is $f(g(x))$ the same as $f(x)$ times $g(x)$?

No. $f(g(x))$ feeds the output of g into f . The product $f(x)g(x)$ multiplies two separate outputs. For $f(x) = x + 1$ and $g(x) = 2x$, $f(g(3)) = 7$ but $f(3)g(3) = 24$.

How do you find the domain of a quotient of functions?

Find the domain shared by f and g , then remove every x where $g(x) = 0$. A zero denominator is never allowed.

How do you decompose a function?

Identify the inside expression and call it $g(x)$, then write the remaining operation as $f(x)$. For $h(x) = \sqrt{x^2 + 7}$, use $g(x) = x^2 + 7$ and $f(x) = \sqrt{x}$.

Can $f(g(x))$ equal $g(f(x))$?

Sometimes, such as when f and g are inverses of each other or both are $f(x) = x + 3$. In general they differ, so always check the order.