

Precalculus - Combining and Composing Functions - Answer Key

Common Core F-BF.A.1b, F-BF.A.1c · maths-pdf.fr/en/

Check each solution here. Every composite is evaluated from the inside out, with domains reasoned step by step and final answers in bold.

Precalculus · Common Core State Standards: **F-BF.A.1b, F-BF.A.1c**

Exercise 1 - Sum of functions at a point

$$f(2) = 4 \text{ and } g(2) = 5.$$

$$(f+g)(2) = 9$$

Exercise 2 - Difference of functions

$$3x-2-(x^2+1) = -x^2+3x-3.$$

$$-x^2+3x-3$$

Exercise 3 - Product of functions

$$(3x-2)(x^2+1) = 3x^3-2x^2+3x-2.$$

$$3x^3-2x^2+3x-2$$

Exercise 4 - Domain of a quotient

$$g(x) = 0 \text{ when } x = \pm 2. \text{ Both } f \text{ and } g \text{ are defined for all reals.}$$

$$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$$

Exercise 5 - Evaluate f of g

$$g(2) = 5. \text{ Then } f(5) = 15 - 2 = 13.$$

Exercise 6 - Evaluate g of f

$$f(2) = 4. \text{ Then } g(4) = 16 + 1 = 17.$$

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Exercise 7 - Composition from a table

x	0	1	2	3
f(x)	2	5	1	4
g(x)	3	0	2	1

Follow each input through g, then f.

$$g(1) = 0, f(0) = 2. f(0) = 2, g(2) = 2. g(3) = 1, f(1) = 5.$$

2, 2, 5

Exercise 8 - Composite formula

$$f(g(x)) = 2(x^2 - 1) + 3 = 2x^2 + 1.$$

$$2x^2 + 1$$

Exercise 9 - Composite in the other order

$$g(f(x)) = (2x + 3)^2 - 1 = 4x^2 + 12x + 9 - 1 = 4x^2 + 12x + 8.$$

$$4x^2 + 12x + 8$$

Exercise 10 - Order matters

$$f(g(x)) = 3x + 5. g(f(x)) = 3(x + 5) = 3x + 15.$$

The results differ because $3x + 5 \neq 3x + 15$

Exercise 11 - Domain of a composite



The domain starts at 4 and continues right.

Require $g(x) = x - 4 \geq 0$.

$[4, \infty)$

Exercise 12 - Two composites with a reciprocal

$f(g(x)) = \frac{1}{x-3}$, domain $x \neq 3$. $g(f(x)) = \frac{1}{x} - 3$, domain $x \neq 0$.

$\frac{1}{x-3}$ with $x \neq 3$; $\frac{1}{x} - 3$ with $x \neq 0$

Exercise 13 - Domain with a square root inside

Require $x^2 - 9 \geq 0$, so $(x-3)(x+3) \geq 0$. The product is nonnegative when $x \leq -3$ or $x \geq 3$.

$(-\infty, -3] \cup [3, \infty)$

Exercise 14 - Decompose a power

The inside expression is $5x - 1$ and the outer operation is the fourth power.

$g(x) = 5x - 1$, $f(x) = x^4$

Exercise 15 - Decompose a root

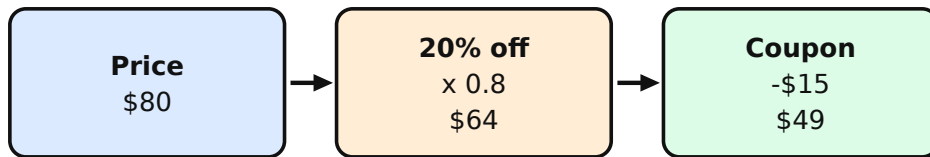
Inside: $x^2 + 7$. Outside: the square root.

$g(x) = x^2 + 7$, $f(x) = \sqrt{x}$

Exercise 16 - Three functions

$h(3) = 9$, $g(9) = 18$, $f(18) = 19$.

Exercise 17 - Coupon and discount

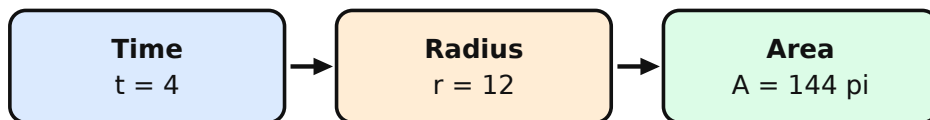


Percent first, then coupon.

$$s(c(80)) = 0.8(65) = 52. \quad c(s(80)) = 0.8(80) - 15 = 64 - 15 = 49.$$

\$52 and \$49; taking the percent off first is cheaper

Exercise 18 - Spreading ripple



At $t = 4$ s.

$$A(r(t)) = \pi(3t)^2 = 9\pi t^2. \quad \text{At } t = 4: 9\pi(16) = 144\pi \approx 452.4.$$

$$9\pi t^2; \text{ about } 452.4 \text{ ft}^2$$

Exercise 19 - Bakery cost model

$$C(n(h)) = 12(8h) + 150 = 96h + 150. \quad \text{For } h = 6: 96(6) + 150 = 726. \quad \text{For } 1206: 96h = 1056, \text{ so } h = 11.$$

$$96h + 150; \$726; 11 \text{ hours}$$

Exercise 20 - Verify two functions undo each other

$$f(g(x)) = \frac{(2x+5)-5}{2} = \frac{2x}{2} = x. \quad g(f(x)) = 2 \cdot \frac{x-5}{2} + 5 = x - 5 + 5 = x.$$

Both compositions equal x