

Precalculus - Functions, Domain and Range - Lesson

Common Core F-IF.A.1, F-IF.A.2, F-IF.B.5 · maths-pdf.fr/en/

A function assigns each input exactly one output, written $f(x)$. The domain is the set of allowed inputs and the range is the set of outputs produced. To find a domain, remove any x that makes a denominator zero or puts a negative number under an even root, then write the result in interval notation.

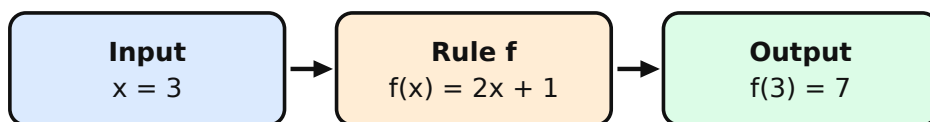
Precalculus · Common Core State Standards: **F-IF.A.1, F-IF.A.2, F-IF.B.5**

Key facts

- A relation is a function only if each input has exactly one output (vertical line test).
- Domain: allowed inputs. Range: outputs that actually occur.
- Exclude values that make a denominator 0 or an even-root radicand negative.
- Interval notation uses brackets for included endpoints and parentheses for excluded ones.
- A piecewise function uses a different rule on each part of its domain.

What is a function?

A relation is a function when every input has exactly one output. Mathematicians write the output of the function f at the input x as $f(x)$. To evaluate, replace every x with the input. For $f(x) = 2x + 1$, $f(3) = 2(3) + 1 = 7$.



A function is a rule: each input goes in once and exactly one output comes out.

Domain and range

The **domain** is the set of all allowed inputs. The **range** is the set of all outputs the function actually produces.

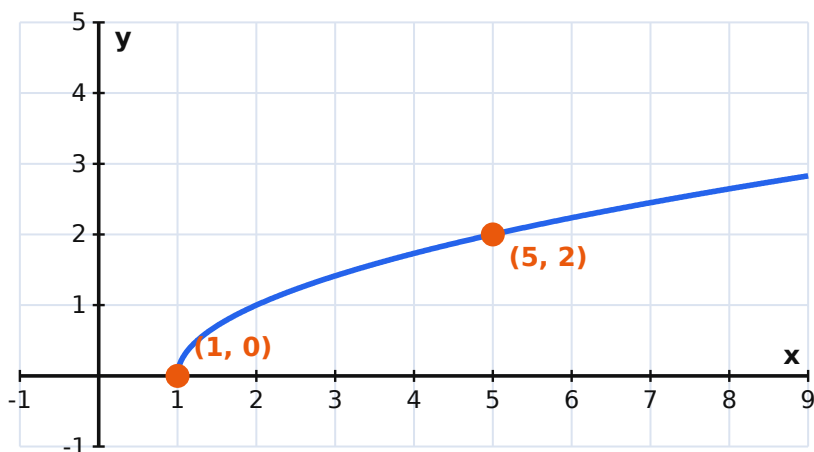
How do you find the domain of a function?

Start with all real numbers, then remove inputs that break a rule of arithmetic. Two rules cover

most of Precalculus:

- You cannot divide by zero, so exclude any x that makes a denominator 0.
- You cannot take an even root of a negative number, so require the radicand to be ≥ 0 .

Example: For $f(x) = \sqrt{x-1}$ we need $x-1 \geq 0$, so $x \geq 1$. The domain is $[1, \infty)$. The smallest output is $f(1) = 0$, so the range is $[0, \infty)$.



The graph of $f(x) = \sqrt{x-1}$ begins at $(1, 0)$: domain $[1, \infty)$, range $[0, \infty)$.

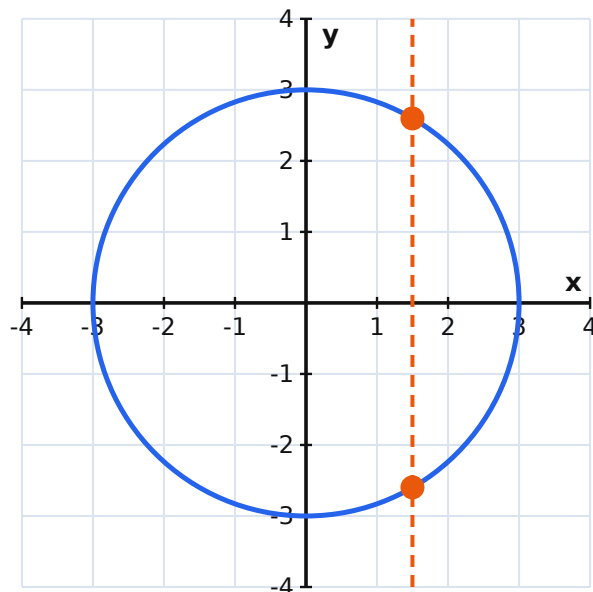
How do you read domain and range from a graph?

The domain is the set of x -values the graph covers from left to right. The range is the set of y -values it covers from bottom to top. A closed dot means the endpoint is included, so use a bracket. An open dot or an arrow past the edge means use a parenthesis.

Set	Interval notation	Inequality
All numbers from 2 to 5, including both	$[2, 5]$	$2 \leq x \leq 5$
Greater than 3	$(3, \infty)$	$x > 3$
All reals except 4	$(-\infty, 4) \cup (4, \infty)$	$x \neq 4$

How do you test whether a graph is a function?

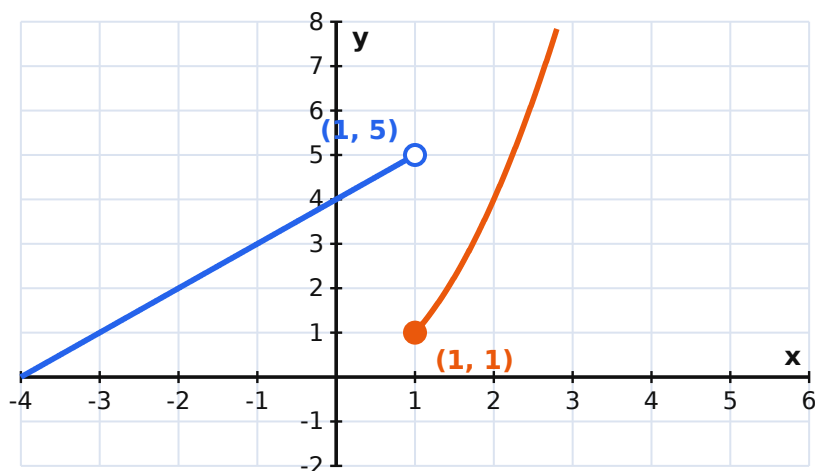
Use the vertical line test: if any vertical line meets the graph in more than one point, one input has two outputs, so the graph is not a function.



The vertical line at $x = 1.5$ hits the circle twice, so a circle is not the graph of a function.

What is a piecewise function?

A piecewise function uses different rules on different parts of its domain. Check which condition the input satisfies, then use only that rule. The graph below uses $y = x + 4$ when $x < 1$ and $y = x^2$ when $x \geq 1$.



An open circle marks an endpoint that is not included; a closed point marks one that is.

Questions about functions, domain and range

What does $f(x)$ mean?

The symbol $f(x)$ means the output of the function f when the input is x . It is not f times x . For

$$f(x) = 3x - 2, f(4) = 3(4) - 2 = 10.$$

How do you find the domain of a rational function?

Set the denominator equal to 0 and solve. Those x-values are excluded. For $1/(x - 4)$, the domain is all real numbers except 4.

What is the difference between domain and range?

The domain lists inputs (x-values) and the range lists outputs (y-values). Domain is read left to right on a graph and range is read bottom to top.

Why does a circle fail the vertical line test?

Most vertical lines cross a circle twice, so one x-value would need two y-values. A function allows only one output per input, so a circle is not a function.

When do you use a bracket and when a parenthesis?

Use a bracket when the endpoint is included, as in $[2, 5]$. Use a parenthesis when the endpoint is excluded or the interval goes to infinity, as in $(3, \text{infinity})$.