

Precalculus - Functions, Domain and Range - Answer Key

Common Core F-IF.A.1, F-IF.A.2, F-IF.B.5 · maths-pdf.fr/en/

Use this key to check each worked solution. Every answer shows the substitution or inequality used, and the final answer is in bold.

Precalculus · Common Core State Standards: **F-IF.A.1, F-IF.A.2, F-IF.B.5**

Exercise 1 - Evaluate a quadratic

Substitute $x = 3$: $2(9) - 5(3) + 1 = 18 - 15 + 1$.

$$f(3) = 4$$

Exercise 2 - Evaluate a cubic

$$g(-2) = (-2)^3 - 4(-2) = -8 + 8.$$

$$g(-2) = 0$$

Exercise 3 - Is it a function?

The input 2 appears twice with different outputs, 5 and 7.

Not a function

Exercise 4 - Domain of a line

A linear function has no division and no roots, so every real number is allowed.

$$(-\infty, \infty)$$

Exercise 5 - Domain with a denominator

Set $x - 4 = 0$, so $x = 4$ is excluded.

$$(-\infty, 4) \cup (4, \infty)$$

Exercise 6 - Domain with a root

Require $2x+6 \geq 0$, so $2x \geq -6$ and $x \geq -3$.

$$[-3, \infty)$$

Exercise 7 - Root and denominator together

The root needs $x+5 \geq 0$, so $x \geq -5$. The denominator excludes $x=2$, which lies inside that interval.

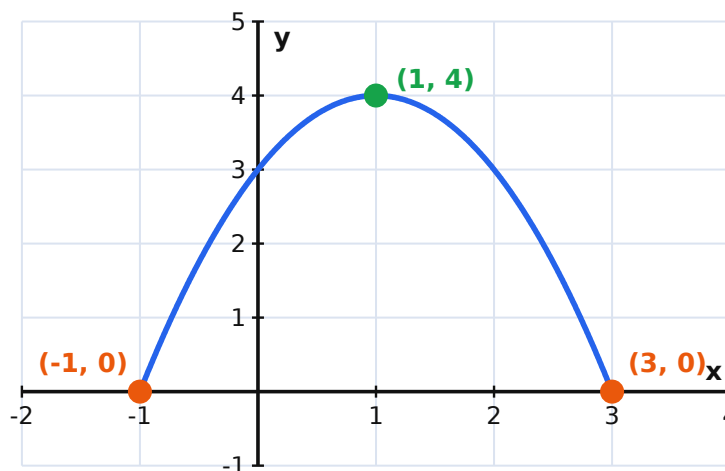
$$[-5, 2) \cup (2, \infty)$$

Exercise 8 - Range of a shifted parabola

Since $x^2 \geq 0$, we have $f(x) \geq 3$, and 3 is reached at $x=0$.

$$[3, \infty)$$

Exercise 9 - Read a graph



Domain runs from $x = -1$ to $x = 3$; the highest point is $(1, 4)$.

The graph runs from $x = -1$ to $x = 3$ with closed endpoints, so the domain is $[-1, 3]$. The lowest y is 0 and the highest is 4, so the range is $[0, 4]$. Using $f(x) = -(x-1)^2 + 4$: $f(2) = -1 + 4 = 3$.

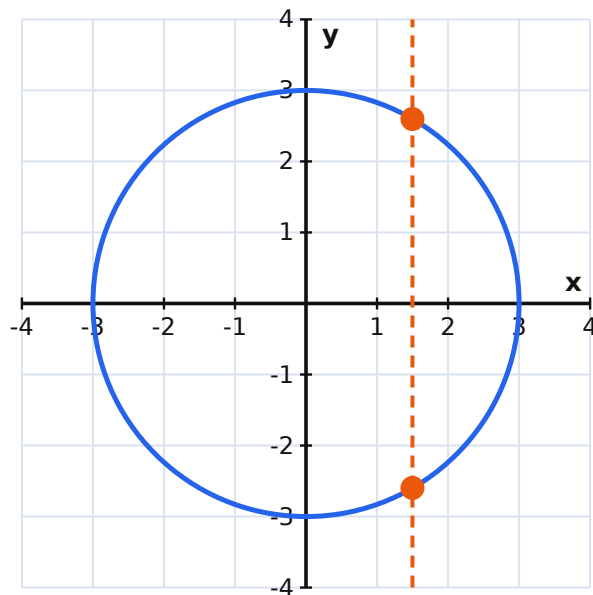
Domain $[-1, 3]$; **range** $[0, 4]$; $f(2) = 3$

Exercise 10 - Piecewise values

$-2 < 1$, so $f(-2) = -2 + 4 = 2$. $1 \geq 1$, so $f(1) = 1^2 = 1$. $3 \geq 1$, so $f(3) = 9$.

2, 1, 9

Exercise 11 - Vertical line test



The vertical line $x = 1.5$ meets the circle at two points.

When $x = 1.5$, $y = \pm\sqrt{9 - 2.25} = \pm\sqrt{6.75} \approx \pm 2.6$. One input gives two outputs.

No, it fails the vertical line test

Exercise 12 - Domain of a model

Both the width w and the length $20 - w$ must be positive, so $w > 0$ and $w < 20$.

$(0, 20)$

Exercise 13 - Reciprocal of a root

The radicand must be positive (it cannot be 0, since that would put 0 in the denominator):
 $x - 3 > 0$.

$(3, \infty)$

Exercise 14 - Solve $f(x) = \text{value}$

$$4x-1=11, \text{ so } 4x=12.$$

$$x=3$$

Exercise 15 - Input for a quadratic output

$$x^2-2x-8=0, \text{ so } (x-4)(x+2)=0.$$

$$x=4 \text{ or } x=-2$$

Exercise 16 - Difference quotient

$$f(2+h)=4+4h+h^2+1=5+4h+h^2 \text{ and } f(2)=5. \text{ The numerator is } 4h+h^2; \text{ divide by } h.$$

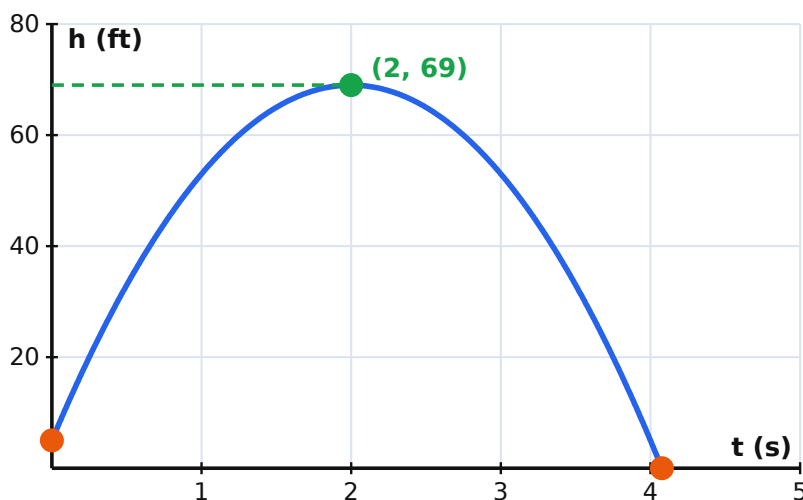
$$4+h$$

Exercise 17 - Taxi fare

$$C(6.4)=4+2.5(6.4)=4+16=20. \text{ For } \$34: 4+2.5m=34, \text{ so } 2.5m=30 \text{ and } m=12.$$

\$20.00; 12 miles

Exercise 18 - Height of a ball



The ball peaks at $(2, 69)$.

$h(0)=5$. $h(1)=-16+64+5=53$. $h(2)=-64+128+5=69$. The vertex is at $t=-\frac{64}{2(-16)}=2$, so the maximum is 69 ft. The ball ends at height 0.

5, 53, 69; **range** $[0,69]$

Exercise 19 - Data plan

For 3 and 5 GB the cost is the flat \$25. For 9.5 GB: $25+8(9.5-5)=25+36=61$. The cost is never below 25 and grows without bound.

\$25, \$25, \$61; **range** $[25,\infty)$

Exercise 20 - Semicircle function

Require $9-x^2 \geq 0$, so $-3 \leq x \leq 3$. The output is at most $f(0)=3$ and at least $f(\pm 3)=0$.

Domain $[-3,3]$; **range** $[0,3]$