

Precalculus - Inverse Functions - Answer Key

Common Core F-BF.B.4a, F-BF.B.4b, F-BF.B.4c, F-BF.B.4d · maths-pdf.fr/en/

Use this key to check each inverse. Solutions show the swap-and-solve steps, the composition check where helpful, and bold final answers.

Precalculus · Common Core State Standards: **F-BF.B.4a, F-BF.B.4b, F-BF.B.4c, F-BF.B.4d**

Exercise 1 - One-to-one table

The output 6 comes from both $x=1$ and $x=3$.

No, it is not one-to-one

Exercise 2 - Inverse of a line

$y = 3x - 6$. Swap: $x = 3y - 6$, so $y = \frac{x+6}{3}$.

$$f^{-1}(x) = \frac{x+6}{3} = \frac{x}{3} + 2$$

Exercise 3 - Inverse of a quotient by a constant

Swap: $x = \frac{y+4}{5}$, so $5x = y + 4$.

$$f^{-1}(x) = 5x - 4$$

Exercise 4 - Inverse of a cubic

Swap: $x = y^3 + 1$, so $y^3 = x - 1$.

$$f^{-1}(x) = \sqrt[3]{x-1}$$

Exercise 5 - Inverse of a rational function

Swap: $x = \frac{2}{y-1}$, so $y-1 = \frac{2}{x}$.

$$f^{-1}(x) = \frac{2}{x} + 1$$

Exercise 6 - Inverse of a root

Swap: $x = \sqrt{y-2}$. Square: $y = x^2 + 2$. Outputs of f are at least 0, so the new domain is $x \geq 0$.

$$f^{-1}(x) = x^2 + 2, x \geq 0$$

Exercise 7 - Inverse of a restricted parabola

Swap: $x = (y-3)^2$. Since $y \geq 3$, $y-3 = \sqrt{x}$.

$$f^{-1}(x) = \sqrt{x} + 3, x \geq 0$$

Exercise 8 - Verify inverses

$$f(g(x)) = 4 \cdot \frac{x-7}{4} + 7 = x. \quad g(f(x)) = \frac{4x+7-7}{4} = x.$$

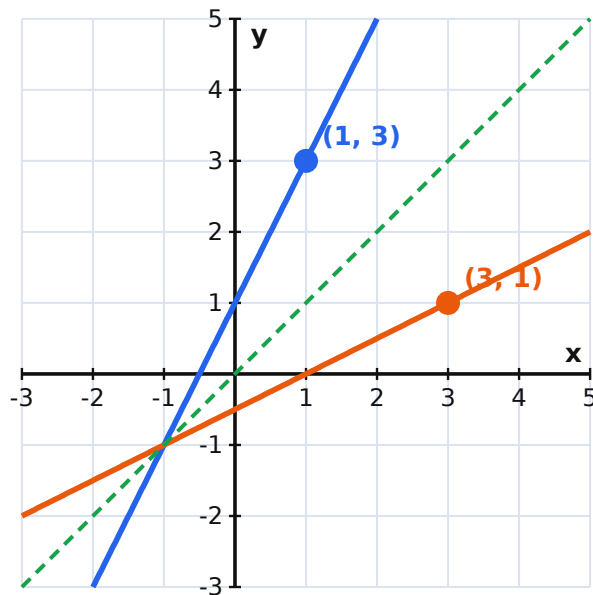
Both compositions equal x

Exercise 9 - Read a table of an inverse

Find the output in the second row: 11 pairs with $x=3$, and 5 with $x=1$. $f^{-1}(14)=4$, so $f(4)=14$.

3, 1, 14

Exercise 10 - Equation from a graph



The orange line is the mirror image of the blue line over $y = x$.

The orange line is the inverse. Solve $x = 2y + 1$ for y .

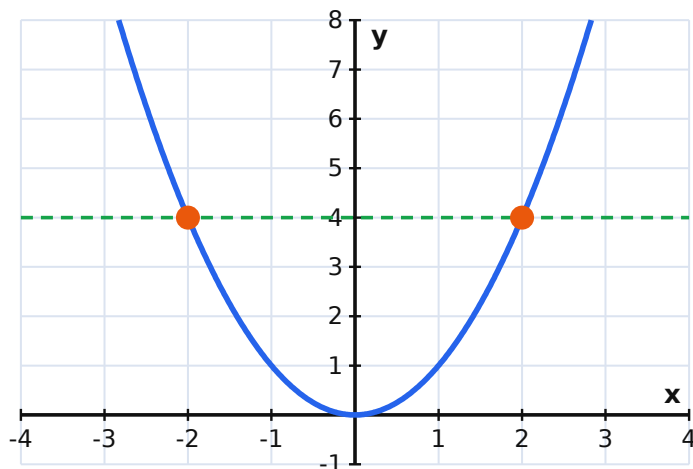
$$y = \frac{x-1}{2}$$

Exercise 11 - Domain and range of an inverse

f : domain $[-1, \infty)$, range $[0, \infty)$. Swap: $x = \sqrt{y+1}$, so $y = x^2 - 1$. The domain and range swap.

$f^{-1}(x) = x^2 - 1$ **with domain** $[0, \infty)$ **and range** $[-1, \infty)$

Exercise 12 - Restrict the domain



A horizontal line hits twice, so restrict to the right half.

Swap: $x = y^2 - 4$, so $y^2 = x + 4$ and $y = \sqrt{x+4}$ because $y \geq 0$.

$$f^{-1}(x) = \sqrt{x+4}, x \geq -4$$

Exercise 13 - Temperature conversion

$F - 32 = 1.8C$, so $C = \frac{F-32}{1.8}$. For $F = 95$: $\frac{63}{1.8} = 35$.

$$C = \frac{F-32}{1.8}; 35^\circ\text{C}$$

Exercise 14 - Rational inverse and check

Swap: $x = \frac{y+2}{y-3}$, so $xy - 3x = y + 2$ and $y(x-1) = 3x+2$. Then $y = \frac{3x+2}{x-1}$. Check: $f(5) = \frac{7}{2}$ and $f^{-1}\left(\frac{7}{2}\right) = \frac{10.5+2}{2.5} = 5$.

$$f^{-1}(x) = \frac{3x+2}{x-1}$$

Exercise 15 - Complete the square first

$y = (x+3)^2 - 9$. Swap: $x = (y+3)^2 - 9$, so $(y+3)^2 = x+9$. Since $y+3 \geq 0$: $y = \sqrt{x+9} - 3$.

$$f^{-1}(x) = \sqrt{x+9} - 3, x \geq -9$$

Exercise 16 - Another rational inverse

Swap: $x = \frac{y}{y+2}$, so $xy + 2x = y$, $y(x-1) = -2x$, $y = \frac{-2x}{x-1}$.

$$f^{-1}(x) = \frac{2x}{1-x}$$

Exercise 17 - Taxi fare inverse

$C - 4 = 2.5m$, so $m = \frac{C-4}{2.5}$. It gives miles for a fare. For $C = 29$: $\frac{25}{2.5} = 10$.

$$m = \frac{C-4}{2.5}; 10 \text{ miles}$$

Exercise 18 - Car value

$V = 24000 - 1800t$, so $t = \frac{24000 - V}{1800}$. For $V = 15000$: $t = \frac{9000}{1800} = 5$. For $V = 12000$: $t = \frac{12000}{1800} = \frac{20}{3} \approx 6.67$.

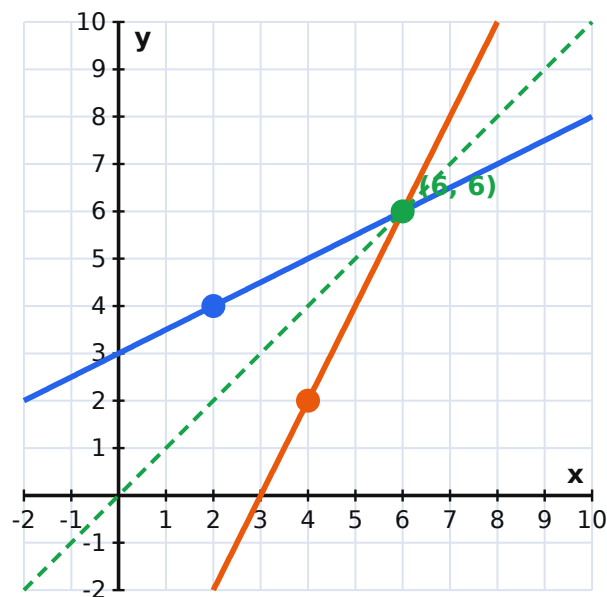
$$t = \frac{24000 - V}{1800}; 5 \text{ years; about } 6.67 \text{ years}$$

Exercise 19 - Find f from its inverse

Swap and solve: $x = \frac{y-3}{2}$ gives $y = 2x + 3$. Check: $f(1) = 5$ and $f^{-1}(5) = \frac{5-3}{2} = 1$.

$$f(x) = 2x + 3$$

Exercise 20 - Where f meets its inverse



f , its inverse, and $y = x$ all meet at $(6, 6)$.

Swap coordinates: $(4, 2)$. Solve $\frac{x}{2} + 3 = x$: $3 = \frac{x}{2}$, so $x = 6$.

$(4, 2)$; $(6, 6)$