

Precalculus - Inverse Functions - Test Answer Key

Common Core F-BF.B.4a, F-BF.B.4b, F-BF.B.4c, F-BF.B.4d · maths-pdf.fr/en/

Worked solutions and scoring notes for the 9-question inverse functions test. Award partial credit using the scoring line under each solution.

Precalculus · Common Core State Standards: **F-BF.B.4a, F-BF.B.4b, F-BF.B.4c, F-BF.B.4d**

Question 1 - Horizontal line test [2 pts]

The horizontal line $y = 4$ meets the graph at $(-2, 4)$ and $(2, 4)$.

No, it is not one-to-one

Scoring: 1 point for the answer, 1 point for the horizontal line reason.

Question 2 - Inverse of a line [2 pts]

Swap: $x = \frac{2y-5}{3}$, so $3x = 2y-5$ and $y = \frac{3x+5}{2}$.

$$f^{-1}(x) = \frac{3x+5}{2}$$

Scoring: 1 point for swapping, 1 point for solving.

Question 3 - Inverse of a root function [2 pts]

$x = \sqrt{y+3}$ gives $y = x^2 - 3$, with $x \geq 0$.

$$f^{-1}(x) = x^2 - 3, x \geq 0$$

Scoring: 1 point for the formula, 1 point for the domain.

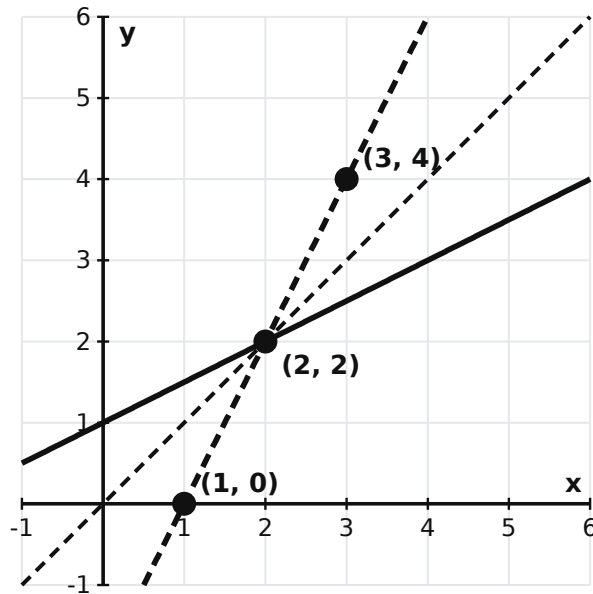
Question 4 - Verify inverses [2 pts]

$$f(g(x)) = 5 \cdot \frac{x+2}{5} - 2 = x. \quad g(f(x)) = \frac{5x-2+2}{5} = x.$$

Both equal x

Scoring: 1 point for each composition.

Question 5 - Points on an inverse [3 pts]



f and f inverse mirror each other over $y = x$.

(a) Swap each pair: $(1,0)$, $(2,2)$, $(3,4)$. (b) $x = \frac{y}{2} + 1$ gives $y = 2x - 2$. (c) They are reflections over the line $y = x$.

$(1,0), (2,2), (3,4)$; $f^{-1}(x) = 2x - 2$; **reflection over** $y = x$

Scoring: 1 point each for (a), (b) and (c).

Question 6 - Rational inverse [2 pts]

Swap: $x(y-2) = y+1$, so $xy - 2x = y+1$, $y(x-1) = 2x+1$.

$$f^{-1}(x) = \frac{2x+1}{x-1}$$

Scoring: 1 point for setting up and clearing the fraction, 1 point for the result.

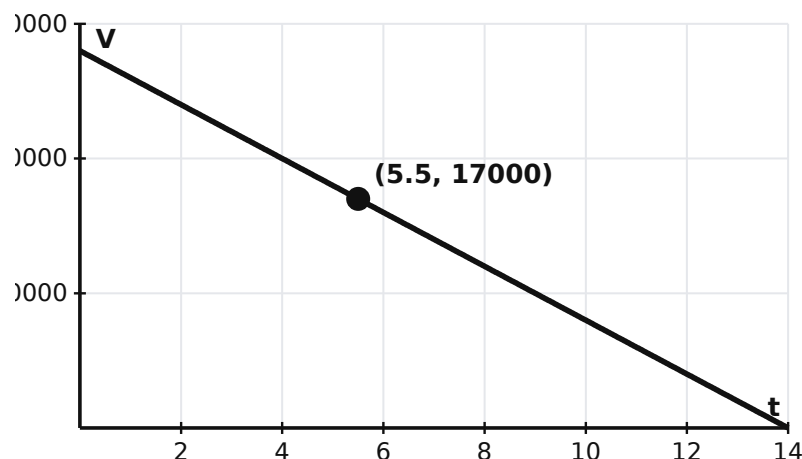
Question 7 - Restricted domain [2 pts]

Swap: $(y+1)^2 = x+4$. Since $y+1 \geq 0$, $y = \sqrt{x+4} - 1$.

$$f^{-1}(x) = \sqrt{x+4} - 1$$

Scoring: 1 point for isolating the square, 1 point for the correct root sign.

Question 8 - Car value model [3 pts]



Value 17,000 at 5.5 years.

(a) $t = \frac{28000 - V}{2000}$. (b) $\frac{11000}{2000} = 5.5$. (c) V^{-1} gives the age of the car for a given value.

$$V^{-1}(V) = \frac{28000 - V}{2000}; 5.5 \text{ years; age from value}$$

Scoring: 1 point for each part.

Question 9 - Domain and range of an inverse [2 pts]

$x - 2 = \sqrt{y - 5}$, so $y = (x - 2)^2 + 5$. Outputs of f are at least 5, so the domain of f^{-1} is $x \geq 5$.

$$f^{-1}(x) = (x - 2)^2 + 5, x \geq 5$$

Scoring: 1 point for the formula, 1 point for the domain.

Total

20 points