

Precalculus - Inverse Trigonometric Functions and Equations - Lesson

Common Core F-TF.B.6, F-TF.B.7 · maths-pdf.fr/en/

Inverse trig functions return angles: $\arcsin x$ is in $[-\pi/2, \pi/2]$, $\arccos x$ is in $[0, \pi]$ and $\arctan x$ is in $(-\pi/2, \pi/2)$. To solve a trig equation, isolate the function, find the reference angle, list every solution in the interval using the quadrants, and add the period ($2\pi k$ or πk) for all solutions.

Precalculus · Common Core State Standards: **F-TF.B.6, F-TF.B.7**

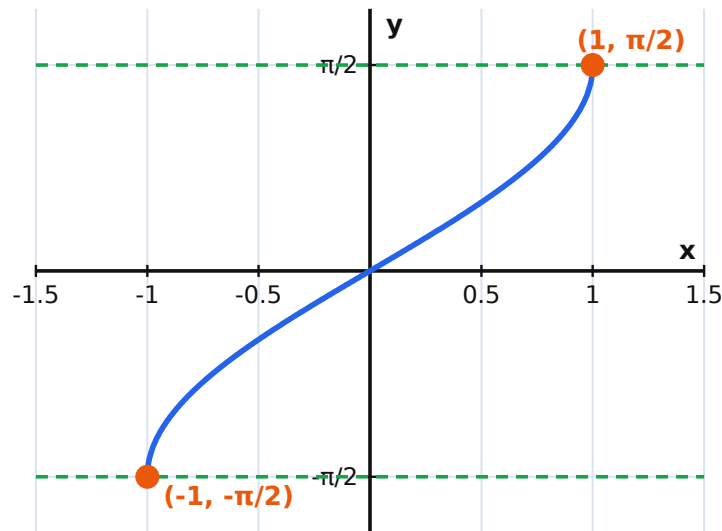
Key facts

- $\arcsin$ and $\arctan$ have range (or closed range) centered on 0; $\arccos$ has range $[0, \pi]$.
- $\sin(\arcsin x) = x$ on $[-1, 1]$, but $\arcsin(\sin x) = x$ only for x in $[-\pi/2, \pi/2]$.
- $\sin^{-1} x$ means the inverse function, not $1/\sin x$.
- Sine and cosine equations usually have two solutions per period; tangent has one.
- Factor quadratic-type trig equations before solving.

What are the inverse trigonometric functions?

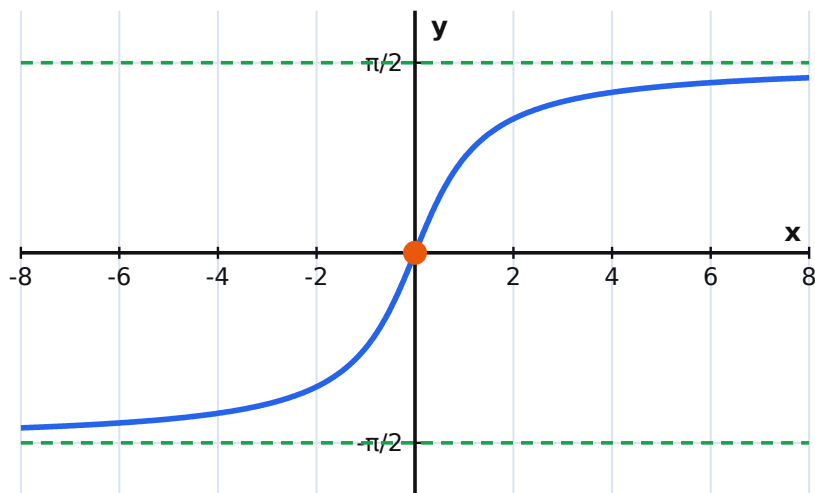
Sine, cosine and tangent are not one-to-one, so each is restricted to an interval where it is. The inverse functions are written $\sin^{-1}x$ or $\arcsin x$, and so on. The notation $\sin^{-1}x$ means the inverse function, not $\frac{1}{\sin x}$.

Function	Domain	Range
$\arcsin x$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\arccos x$	$[-1, 1]$	$[0, \pi]$
$\arctan x$	all real numbers	$(-\frac{\pi}{2}, \frac{\pi}{2})$



$y = \arcsin x$: the output is an angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

To evaluate, ask: which angle in the range has this value? $\arcsin \frac{1}{2} = \frac{\pi}{6}$, $\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$, and $\arctan \sqrt{3} = \frac{\pi}{3}$.



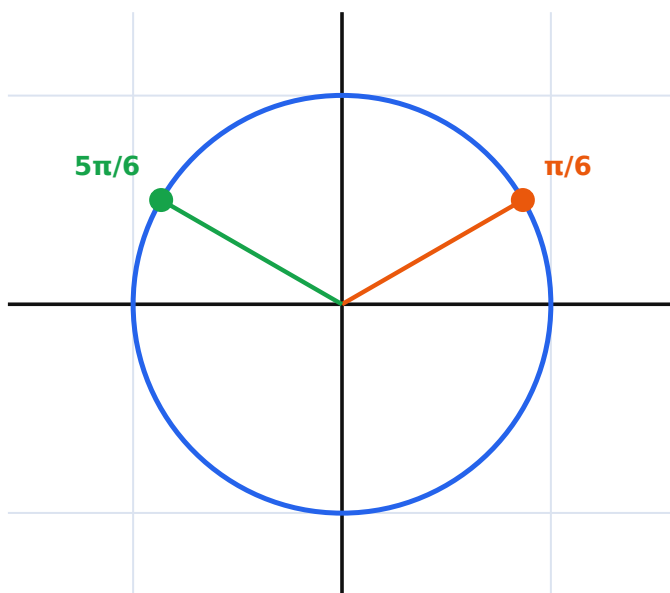
$y = \arctan x$ has horizontal asymptotes $y = \pm \frac{\pi}{2}$.

How do you evaluate compositions?

$\sin(\arcsin x) = x$ for x in $[-1, 1]$. But $\arcsin(\sin x) = x$ only when x is in $[-\frac{\pi}{2}, \frac{\pi}{2}]$. For example $\arcsin(\sin \frac{7\pi}{6}) = -\frac{\pi}{6}$, because the answer must be in the range. For mixed compositions, draw a right triangle: if $\theta = \arcsin \frac{3}{5}$, then $\sin \theta = \frac{3}{5}$ and the third side is 4, so $\cos \theta = \frac{4}{5}$.

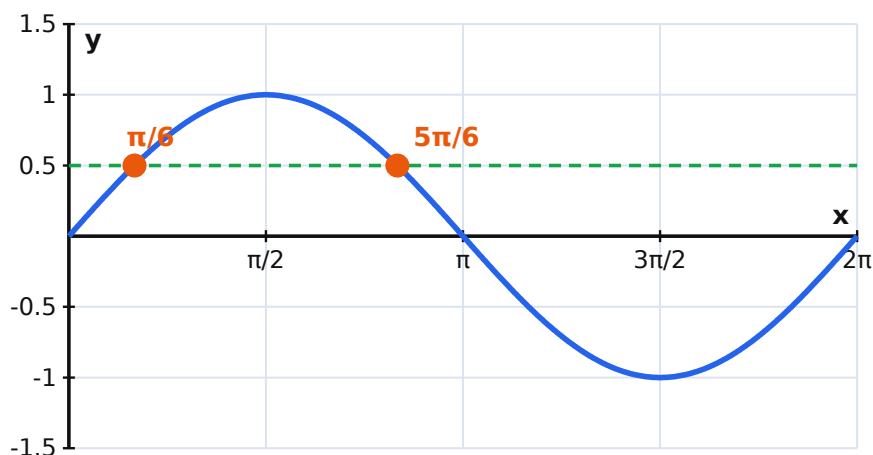
How do you solve a trigonometric equation?

1. Isolate the trig function, for example $\sin x = \frac{1}{2}$.
2. Find the reference angle with an inverse function or the unit circle.
3. Use the quadrants to list all solutions in the interval.
4. For all solutions, add the period: $2\pi k$ for sine and cosine, πk for tangent.



$\sin x = \frac{1}{2}$ has two solutions in $[0, 2\pi)$, in quadrants I and II.

If an equation is quadratic in a trig function, factor it first. For $2\sin^2 x - \sin x - 1 = 0$, factor $(2\sin x + 1)(\sin x - 1) = 0$, so $\sin x = -\frac{1}{2}$ or $\sin x = 1$, giving $x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$.



Graphically, each solution is an intersection of the sine curve with the horizontal line.

Inverse trig functions also turn side lengths into angles. In a right triangle with opposite side a , adjacent side b and hypotenuse c , the acute angle θ satisfies $\theta = \arcsin \frac{a}{c} = \arccos \frac{b}{c} = \arctan \frac{a}{b}$. A ramp that rises 2 feet over a 12-foot length makes an angle of $\arcsin \frac{2}{12} \approx 9.6^\circ$. This is how surveyors, builders and pilots find angles from measured distances.

When the value is not a special angle, use a calculator in radian mode. For $\cos x = 0.3$, $x = \arccos 0.3 \approx 1.266$ and the second solution is $2\pi - 1.266 \approx 5.017$.

Questions about inverse trigonometric functions and equations

Why are the ranges of inverse trig functions restricted?

A function needs one output for each input. Restricting sine, cosine and tangent to intervals where they are one-to-one gives each inverse exactly one answer.

Is $\sin^{-1} x$ the same as $1/\sin x$?

No. The exponent -1 on a function name means inverse. The reciprocal $1/\sin x$ is written $(\sin x)^{-1}$ or $\csc x$.

Why is $\arcsin(\sin(7\pi/6))$ not $7\pi/6$?

The output of $\arcsin$ must lie in $[-\pi/2, \pi/2]$. Since $\sin(7\pi/6) = -1/2$, $\arcsin(-1/2) = -\pi/6$, which is in that interval.

How many solutions does $\sin x = 1/2$ have?

In one period $[0, 2\pi)$ it has two: $\pi/6$ and $5\pi/6$. On all real numbers it has infinitely many, written $\pi/6 + 2\pi k$ and $5\pi/6 + 2\pi k$.

Should the calculator be in degree or radian mode?

Match the problem. If angles are given in radians, use radian mode. A mode mismatch is a common cause of wrong answers.