

Precalculus - Inverse Trigonometric Functions and Equations - Answer Key

Common Core F-TF.B.6, F-TF.B.7 · maths-pdf.fr/en/

Use this key to check each inverse trig value and equation. Solutions show the reference angle, the quadrants and the period, with final answers in bold.

Precalculus · Common Core State Standards: **F-TF.B.6, F-TF.B.7**

Exercise 1 - Exact values of inverse trig

$$\sin \frac{\pi}{6} = \frac{1}{2}. \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}. \tan \frac{\pi}{4} = 1.$$

$$\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{4}$$

Exercise 2 - Negative inputs

Arcsin and arctan give angles in $(-\frac{\pi}{2}, \frac{\pi}{2})$: $-\frac{\pi}{4}$ and $-\frac{\pi}{3}$. Arccos gives an angle in $[0, \pi]$, here quadrant II: $\frac{2\pi}{3}$.

$$-\frac{\pi}{4}, \frac{2\pi}{3}, -\frac{\pi}{3}$$

Exercise 3 - Domain and range

The graph runs from $x = -1$ to $x = 1$, and y from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

Domain $[-1, 1]$; **range** $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Exercise 4 - Calculator values

$$\arcsin 0.4 \approx 0.4115. \arctan 2.5 \approx 1.1903.$$

0.4115 and 1.1903

Exercise 5 - Inverse properties

Each function undoes its inverse, and all the inputs are in the right domain or range.

$0.3, -0.8, \frac{\pi}{6}$

Exercise 6 - Outside the principal range

$$\sin \frac{7\pi}{6} = -\frac{1}{2} \text{ and } \arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}, \cos\left(-\frac{\pi}{3}\right) = \frac{1}{2} \text{ and } \arccos \frac{1}{2} = \frac{\pi}{3}.$$

$$-\frac{\pi}{6} \text{ and } \frac{\pi}{3}$$

Exercise 7 - Cosine of an arcsine

Let $\theta = \arcsin \frac{3}{5}$, an angle in quadrant I. The triangle has opposite 3, hypotenuse 5, so adjacent 4.

$$\frac{4}{5}$$

Exercise 8 - Tangent of an arccosine

Adjacent 5, hypotenuse 13, so opposite $\sqrt{169-25}=12$. $\tan \theta = \frac{12}{5}$.

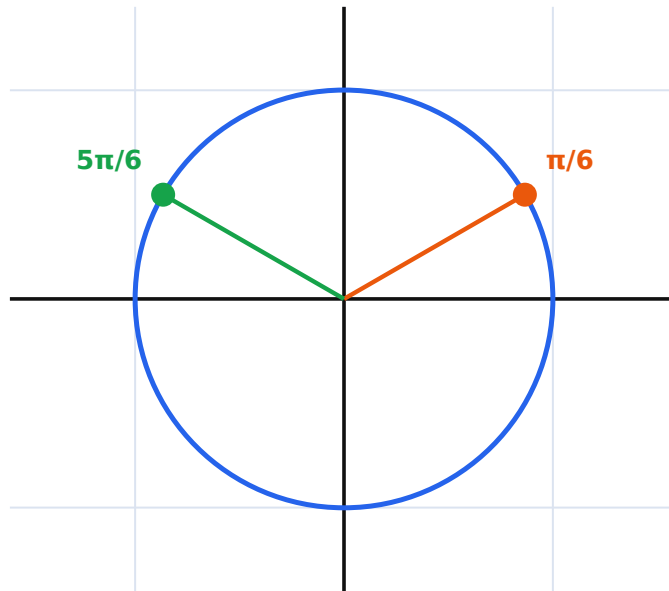
$$\frac{12}{5}$$

Exercise 9 - Sine of an arctangent

Opposite 3, adjacent 4, hypotenuse 5.

$$\frac{3}{5}$$

Exercise 10 - Solve $\sin x = 1/2$



Quadrants I and II.

The reference angle is $\frac{\pi}{6}$. Sine is positive in quadrants I and II: $\frac{\pi}{6}$ and $\pi - \frac{\pi}{6}$.

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

Exercise 11 - Solve $\cos x$

Reference angle $\frac{\pi}{4}$. Cosine is negative in quadrants II and III: $\pi - \frac{\pi}{4}$ and $\pi + \frac{\pi}{4}$.

$$x = \frac{3\pi}{4}, \frac{5\pi}{4}$$

Exercise 12 - Solve $\tan x$

Reference angle $\frac{\pi}{3}$. Tangent is negative in quadrants II and IV: $\pi - \frac{\pi}{3}$ and $2\pi - \frac{\pi}{3}$.

$$x = \frac{2\pi}{3}, \frac{5\pi}{3}$$

Exercise 13 - General solutions

In one period: $\frac{\pi}{3}$ and $\frac{2\pi}{3}$. Add $2\pi k$.

$$x = \frac{\pi}{3} + 2\pi k \text{ or } x = \frac{2\pi}{3} + 2\pi k, k \text{ an integer}$$

Exercise 14 - Rearrange first

$\sin x = \frac{\sqrt{2}}{2}$. Reference angle $\frac{\pi}{4}$, quadrants I and II.

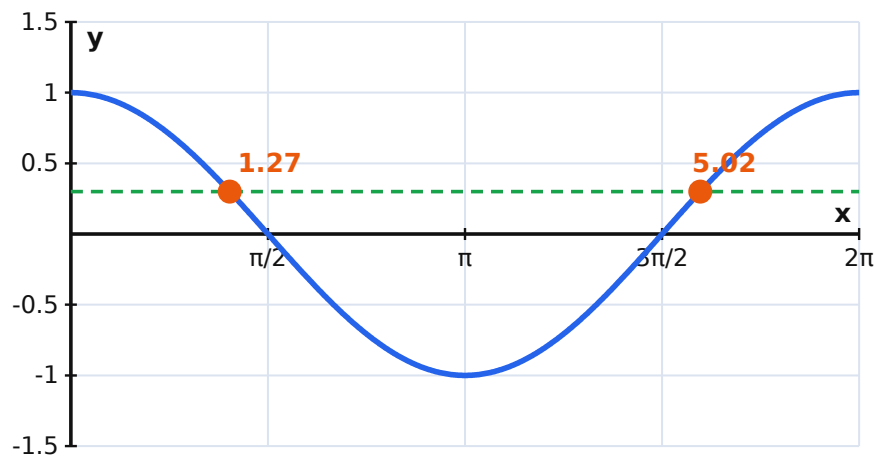
$$x = \frac{\pi}{4}, \frac{3\pi}{4}$$

Exercise 15 - Quadratic in sine

Factor: $(2\cos x - 1)(\cos x + 1) = 0$. $\cos x = \frac{1}{2}$ gives $\frac{\pi}{3}$ and $\frac{5\pi}{3}$. $\cos x = -1$ gives π .

$$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

Exercise 16 - A non-special value



Two intersections.

$x = \arccos 0.3 \approx 1.266$. The other solution is $2\pi - 1.266 \approx 5.017$.

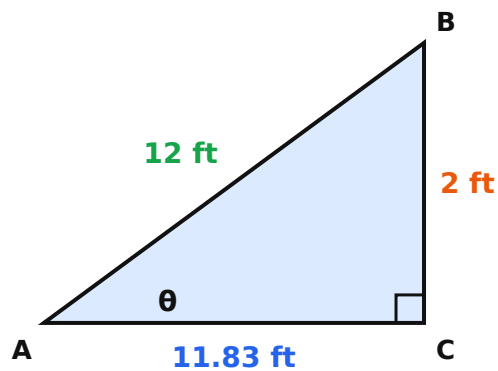
$x \approx 1.266$ and 5.017

Exercise 17 - Double angle inside

Let $u = 2x$ on $[0, 4\pi)$. $\sin u = \frac{\sqrt{3}}{2}$ gives $u = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$. Divide by 2.

$$x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$$

Exercise 18 - Ramp angle

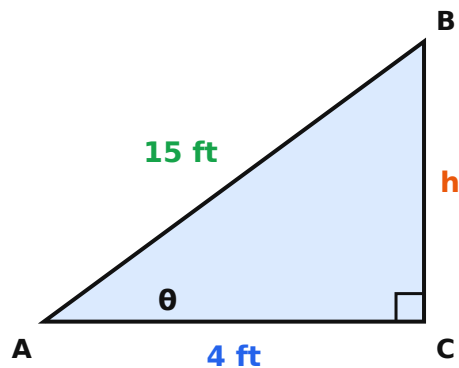


$$\sin \theta = 2/12.$$

$$\sin \theta = \frac{2}{12} = \frac{1}{6}, \text{ so } \theta = \arcsin \frac{1}{6} \approx 9.6^\circ.$$

$$9.6^\circ$$

Exercise 19 - Ladder



$$\cos \theta = 4/15.$$

$$\cos \theta = \frac{4}{15}, \text{ so } \theta = \arccos \frac{4}{15} \approx 74.5^\circ.$$

$$74.5^\circ$$

Exercise 20 - Ferris wheel timing

$30 - 25 \cos u = 40$, so $\cos u = -0.4$ where $u = \frac{\pi}{15}$. $u = \arccos(-0.4) \approx 1.9823$, so $t \approx 9.46$. The other solution in a cycle is $u = 2\pi - 1.9823$, so $t = 30 - 9.46 = 20.54$.

$$t \approx 9.46 \text{ s and } 20.54 \text{ s}$$