

Precalculus - Law of Sines and Law of Cosines - Lesson

Common Core G-SRT.D.10, G-SRT.D.11, G-SRT.C.8 · maths-pdf.fr/en/

The Law of Sines, $a/\sin A = b/\sin B = c/\sin C$, solves triangles given two angles and a side, or two sides and an opposite angle. The Law of Cosines, $c^2 = a^2 + b^2 - 2ab \cos C$, solves SAS and SSS triangles. Area is $1/2 ab \sin C$, or Heron's formula for three sides.

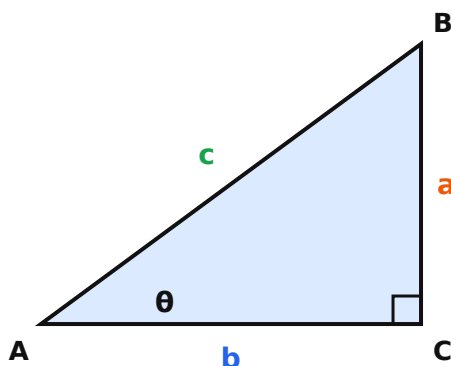
Precalculus · Common Core State Standards: **G-SRT.D.10, G-SRT.D.11, G-SRT.C.8**

Key facts

- Law of Sines: $a/\sin A = b/\sin B = c/\sin C$.
- Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$.
- SSA can give 0, 1 or 2 triangles (the ambiguous case).
- Area = $(1/2) ab \sin C$; Heron: $\sqrt{s(s-a)(s-b)(s-c)}$ with $s = (a+b+c)/2$.
- The angles of a triangle add to 180 degrees; the largest angle is opposite the longest side.

How do you solve a right triangle?

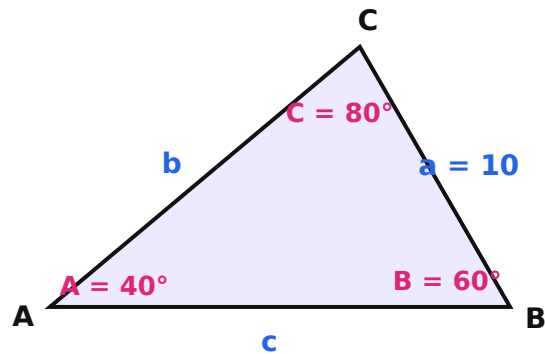
In a right triangle with acute angle θ , $\sin \theta = \frac{\text{opp}}{\text{hyp}}$, $\cos \theta = \frac{\text{adj}}{\text{hyp}}$ and $\tan \theta = \frac{\text{opp}}{\text{adj}}$. For triangles without a right angle, use two laws. In triangle ABC , side a is opposite angle A , and so on.



Right triangle: $\sin \theta = \frac{a}{c}$, $\cos \theta = \frac{b}{c}$, $\tan \theta = \frac{a}{b}$.

Law of Sines and Law of Cosines

Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$.



Triangle with $A = 40^\circ$, $B = 60^\circ$ and $a = 10$: the Law of Sines gives b and c .

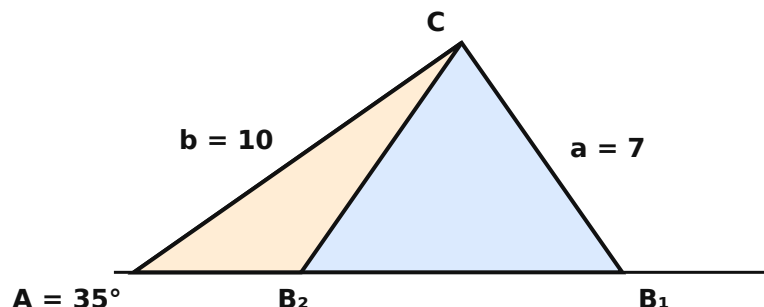
When do you use each law?

Known parts	Use
Two angles and any side (AAS or ASA)	Law of Sines
Two sides and a non-included angle (SSA)	Law of Sines (check for 0, 1 or 2 triangles)
Two sides and the included angle (SAS)	Law of Cosines
Three sides (SSS)	Law of Cosines (to find an angle)

Example (Law of Sines): With $A = 40^\circ$, $B = 60^\circ$ and $a = 10$: $b = \frac{10 \sin 60^\circ}{\sin 40^\circ} \approx 13.47$, $C = 80^\circ$, $c = \frac{10 \sin 80^\circ}{\sin 40^\circ} \approx 15.32$.

What is the ambiguous case?

When you know two sides and an angle opposite one of them (SSA), there can be no triangle, one triangle or two triangles. Find $\sin B = \frac{b \sin A}{a}$. If it is greater than 1, there is no triangle. If it is less than 1, then B and $180^\circ - B$ are both possible; keep each one only if its angle sum with A is less than 180° .



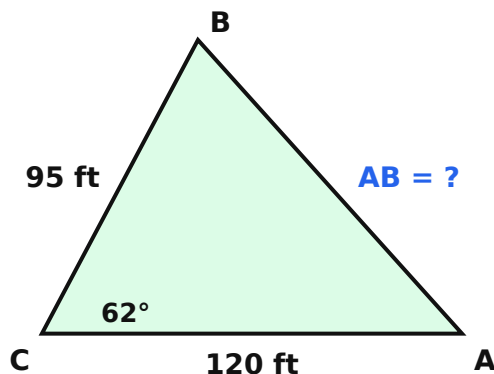
With $A = 35^\circ$, $b = 10$ and $a = 7$, side a can reach the base in two places.

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How do you find the area of a triangle?

When two sides and the included angle are known, $\text{Area} = \frac{1}{2}ab\sin C$. When only the three sides are known, use Heron's formula: with semiperimeter $s = \frac{a+b+c}{2}$, $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$. For sides 13, 14, 15, $s = 21$ and the area is $\sqrt{21 \cdot 8 \cdot 7 \cdot 6} = 84$.



Surveying: the Law of Cosines gives the distance AB across a pond.

For the pond, $AB^2 = 120^2 + 95^2 - 2(120)(95)\cos 62^\circ \approx 12,721$, so $AB \approx 112.8$ feet. Always sketch the triangle first, label known parts, and check that the largest angle faces the longest side.

Questions about law of sines and law of cosines

When should I use the Law of Cosines?

Use it when you know two sides and the included angle (SAS) or all three sides (SSS). In both cases the Law of Sines cannot start because it needs an angle-side pair.

What is the ambiguous case?

It happens with SSA information. The given side opposite the given angle may be too short to reach (no triangle), just reach (one right triangle), or swing to two different positions (two triangles).

How do I know if there are two triangles?

Compute $\sin B$. If it is less than 1, find B and 180 degrees minus B. If the second angle plus the given angle A is under 180 degrees, both triangles exist.

What is Heron's formula used for?

It gives the area of a triangle from its three side lengths only. Compute s, the half perimeter, then take the square root of $s(s-a)(s-b)(s-c)$.

Does the Law of Cosines include the Pythagorean theorem?

Yes. When $C = 90$ degrees, $\cos C = 0$ and the law becomes $c^2 = a^2 + b^2$.