

Precalculus - Law of Sines and Law of Cosines - Answer Key

Common Core G-SRT.D.10, G-SRT.D.11, G-SRT.C.8 · maths-pdf.fr/en/

Use this key to check each triangle. Solutions show which law is used, the setup and the rounded result, with final answers in bold.

Precalculus · Common Core State Standards: **G-SRT.D.10, G-SRT.D.11, G-SRT.C.8**

Exercise 1 - Right triangle: find a leg

$$\sin 35^\circ = \frac{x}{20}, \text{ so } x = 20 \sin 35^\circ \approx 11.47.$$

11.47

Exercise 2 - Right triangle: tangent

$$\tan 62^\circ = \frac{h}{8}, \text{ so } h = 8 \tan 62^\circ \approx 15.05.$$

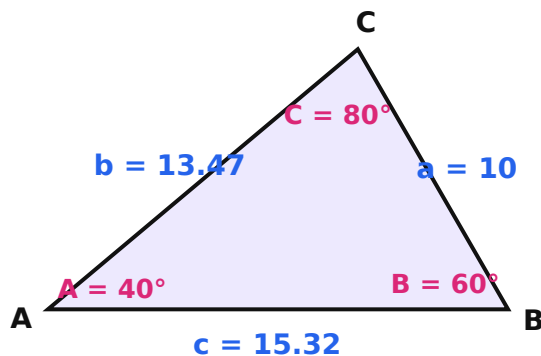
15.05 ft

Exercise 3 - Solve a right triangle

$$c = \sqrt{49 + 576} = 25. \arctan \frac{7}{24} \approx 16.26^\circ, \text{ and the other angle is } 90^\circ - 16.26^\circ = 73.74^\circ.$$

25; 16.26° and 73.74°

Exercise 4 - Law of Sines (AAS)



Solved triangle.

$$C = 180^\circ - 40^\circ - 60^\circ = 80^\circ. \quad b = \frac{10 \sin 60^\circ}{\sin 40^\circ} \approx 13.47. \quad c = \frac{10 \sin 80^\circ}{\sin 40^\circ} \approx 15.32.$$

$$C = 80^\circ, \quad b \approx 13.47, \quad c \approx 15.32$$

Exercise 5 - Law of Sines (ASA-like)

$$B = 60^\circ. \quad a = \frac{12 \sin 50^\circ}{\sin 70^\circ} \approx 9.78. \quad b = \frac{12 \sin 60^\circ}{\sin 70^\circ} \approx 11.06.$$

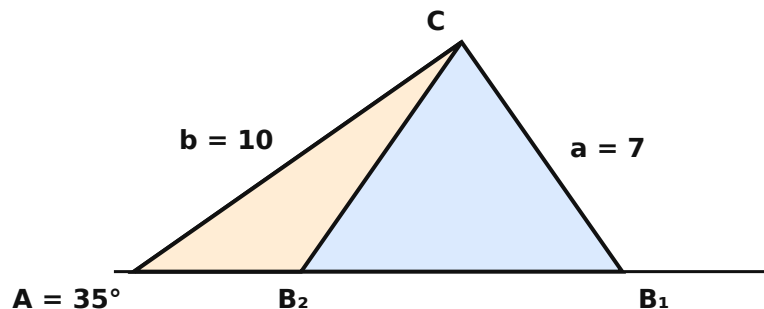
$$B = 60^\circ, \quad a \approx 9.78, \quad b \approx 11.06$$

Exercise 6 - SSA with one triangle

$$\sin B = \frac{10 \sin 40^\circ}{15} \approx 0.4285, \text{ so } B \approx 25.37^\circ. \text{ The other value } 180^\circ - 25.37^\circ \text{ is too large since } A + B > 180^\circ. \quad C \approx 114.63^\circ, \quad c = \frac{15 \sin C}{\sin 40^\circ} \approx 21.21.$$

$$B \approx 25.37^\circ, \quad C \approx 114.63^\circ, \quad c \approx 21.21$$

Exercise 7 - Ambiguous case: two triangles



B1 and B2.

$$\sin B = \frac{10 \sin 35^\circ}{7} \approx 0.8194. \text{ So } B \approx 55.0^\circ \text{ or } 180^\circ - 55.0^\circ = 125.0^\circ. \text{ Both leave a positive third angle (} 35 + 125.0 < 180 \text{).}$$

Two triangles: $B \approx 55.0^\circ$ or 125.0°

Exercise 8 - No triangle

$$\sin B = \frac{10 \sin 50^\circ}{5} \approx 1.532 > 1.$$

No triangle exists

Exercise 9 - Law of Cosines (SAS)

$$c^2 = 64 + 121 - 2(8)(11)\cos 48^\circ \approx 67.23, \text{ so } c \approx 8.20.$$

$$c \approx 8.20$$

Exercise 10 - Law of Cosines (SSS)

$$\text{The largest angle is opposite 12: } \cos C = \frac{49 + 81 - 144}{2 \cdot 7 \cdot 9} = -\frac{14}{126} \approx -0.1111, \text{ so } C \approx 96.38^\circ.$$

$$96.38^\circ$$

Exercise 11 - All three angles

$$\text{Angle opposite 5: } \cos A = \frac{36 + 49 - 25}{84}, A \approx 44.42^\circ. \text{ Angle opposite 6: } \cos B = \frac{25 + 49 - 36}{70}, B \approx 57.12^\circ. \\ C = 180^\circ - 44.42^\circ - 57.12^\circ = 78.46^\circ.$$

$$44.42^\circ, 57.12^\circ, 78.46^\circ$$

Exercise 12 - Area from two sides

$$\frac{1}{2}(9)(14)\sin 35^\circ \approx 36.14.$$

$$36.14 \text{ square units}$$

Exercise 13 - Heron's formula

$$s = \frac{13 + 14 + 15}{2} = 21. \text{ Area} = \sqrt{21 \cdot 8 \cdot 7 \cdot 6} = \sqrt{7056}.$$

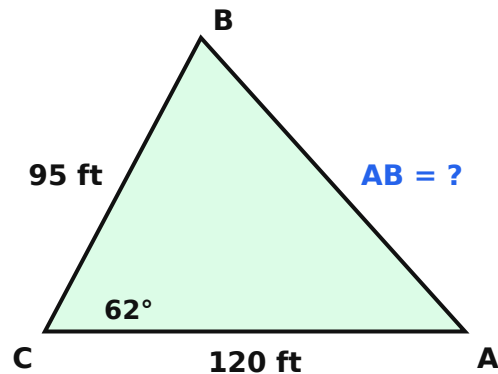
$$84 \text{ square units}$$

Exercise 14 - Choose the law

(a) SAS: Law of Cosines. (b) Two angles and a side: Law of Sines. (c) SSS: Law of Cosines. (d) SSA with a pair α, A : Law of Sines.

(a) Cosines; (b) Sines; (c) Cosines; (d) Sines

Exercise 15 - Width of a pond

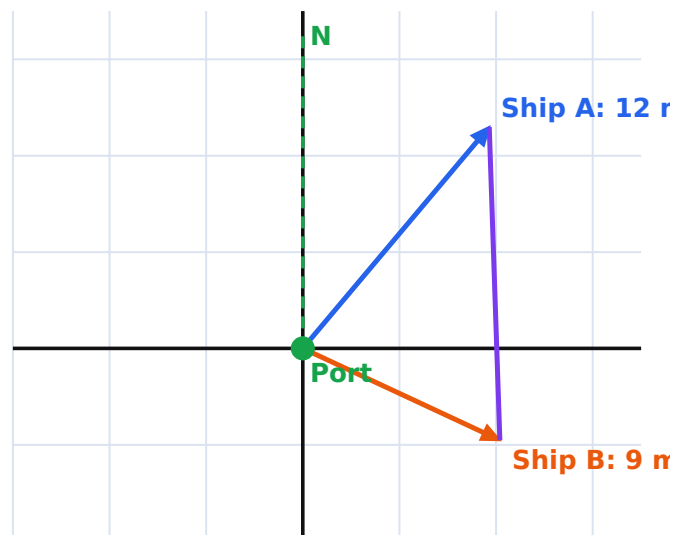


AB is opposite the 62 degree angle.

$$AB^2 = 120^2 + 95^2 - 2(120)(95)\cos 62^\circ \approx 12721, \text{ so } AB \approx 112.8.$$

112.8 ft

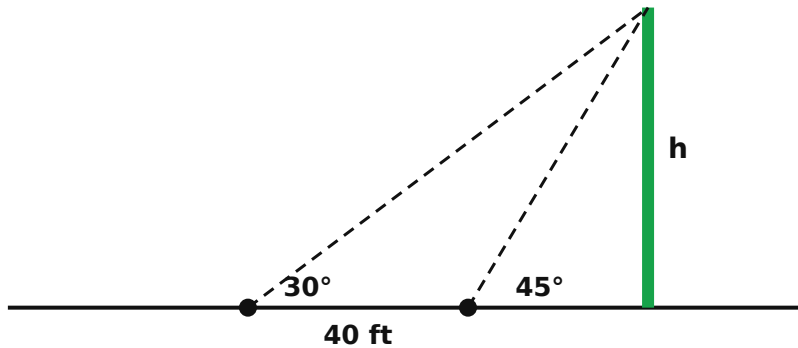
Exercise 16 - Two ships



The angle at port is $115 - 40 = 75$ degrees.

The angle at the port is $115^\circ - 40^\circ = 75^\circ$. $d^2 = 144 + 81 - 2(12)(9)\cos 75^\circ \approx 169.1$, so $d \approx 13.0$.
13.0 mi

Exercise 17 - Height of a tree



Distance from the base: $h/\tan 30$ and $h/\tan 45$.

The distances to the base are $\frac{h}{\tan 30^\circ} = h\sqrt{3}$ and $\frac{h}{\tan 45^\circ} = h$. Their difference is 40: $h(\sqrt{3}-1) = 40$, so $h = \frac{40}{\sqrt{3}-1} \approx 54.6$.

54.6 ft

Exercise 18 - Triangular garden

$s = 36$. Area = $\sqrt{36 \cdot 18 \cdot 12 \cdot 6} = 216$. Since $18^2 + 24^2 = 900 = 30^2$, the largest angle is a right angle.

216 ft²; 90°

Exercise 19 - Roof truss

$\cos C = \frac{100+100-144}{200} = 0.28$, so $C \approx 73.74^\circ$. The base angles are equal: $\frac{180^\circ - 73.74^\circ}{2} = 53.13^\circ$.

Apex 73.74° ; base angles 53.13°

Exercise 20 - Area from angles and a side

$C = 70^\circ$. $b = \frac{10 \sin 70^\circ}{\sin 40^\circ} \approx 14.62$. Area = $\frac{1}{2}ab \sin C = \frac{1}{2}(10)(14.62)\sin 70^\circ \approx 68.7$.

68.7 square units