

Precalculus - Introduction to Limits - Lesson

Common Core F-IF.B.6, F-IF.C.7d · maths-pdf.fr/en/

A limit describes the value a function approaches as x gets close to a number a . The average rate of change is the slope of a secant line, and the limit of average rates as the interval shrinks gives the instantaneous rate of change. Evaluate limits by substitution, factoring or rationalizing, and check that the left and right limits agree.

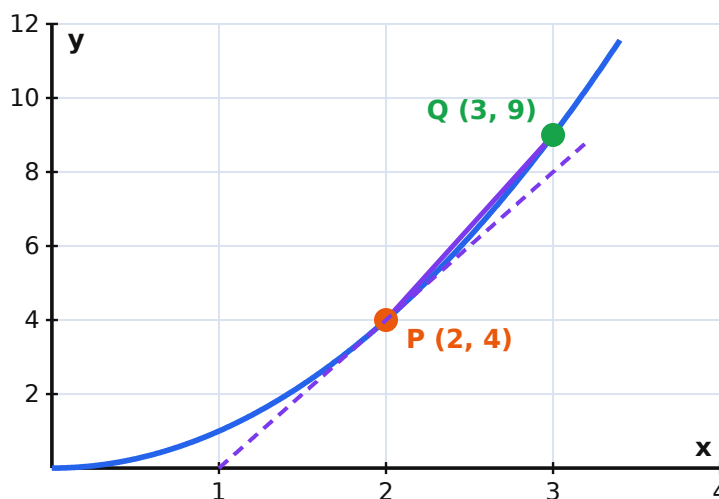
Precalculus · Common Core State Standards: **F-IF.B.6, F-IF.C.7d**

Key facts

- Average rate of change: $(f(b) - f(a))/(b - a)$, the secant slope.
- $\lim_{x \rightarrow a} f(x) = L$ means $f(x)$ approaches L as x approaches a from both sides.
- The limit can exist even if $f(a)$ is undefined (a hole).
- Two-sided limit exists only if the left and right limits are equal.
- f is continuous at a if $f(a)$ exists, the limit exists and they are equal.

What is the average rate of change?

The average rate of change of f from $x=a$ to $x=b$ is $\frac{f(b)-f(a)}{b-a}$, the slope of the secant line through $(a, f(a))$ and $(b, f(b))$. For $f(x) = x^2$ from 2 to 3: $\frac{9-4}{1} = 5$. As the second point slides toward the first, the secant line becomes the tangent line, and its slope gives the **instantaneous rate of change**.



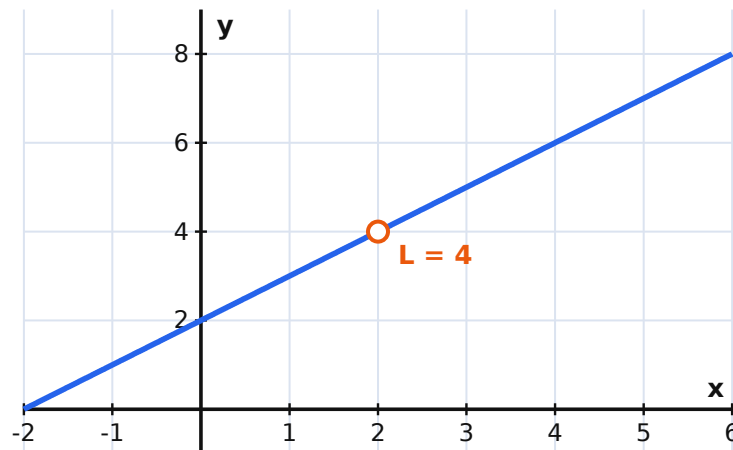
The secant slope 5 from $x = 2$ to $x = 3$ is close to the tangent slope 4 at $x = 2$.

What is a limit?

Limit

$\lim_{x \rightarrow a} f(x) = L$ means that $f(x)$ gets arbitrarily close to L as x gets close to a , from both sides, whether or not $f(a)$ exists or equals L .

The limit depends on the behavior near a , not at a . For $f(x) = \frac{x^2-4}{x-2}$, $f(2)$ is undefined, yet for $x \neq 2$, $f(x) = x+2$, so $\lim_{x \rightarrow 2} f(x) = 4$.



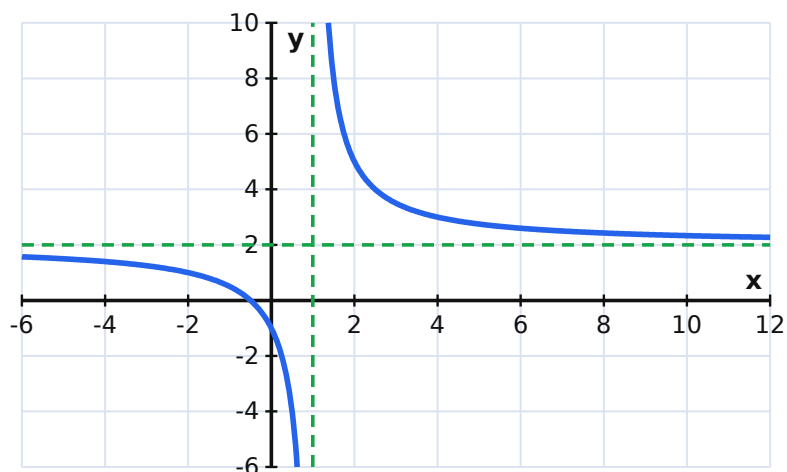
A hole in the graph does not prevent the limit from existing.

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	3.9	3.99	3.999	4.001	4.01	4.1

How do you evaluate limits algebraically?

1. Try direct substitution. If f is a polynomial or rational function defined at a ,
 $\lim_{x \rightarrow a} f(x) = f(a)$.
2. If you get $\frac{0}{0}$, factor and cancel, or rationalize a root.
3. For limits at infinity, divide by the highest power of x in the denominator.

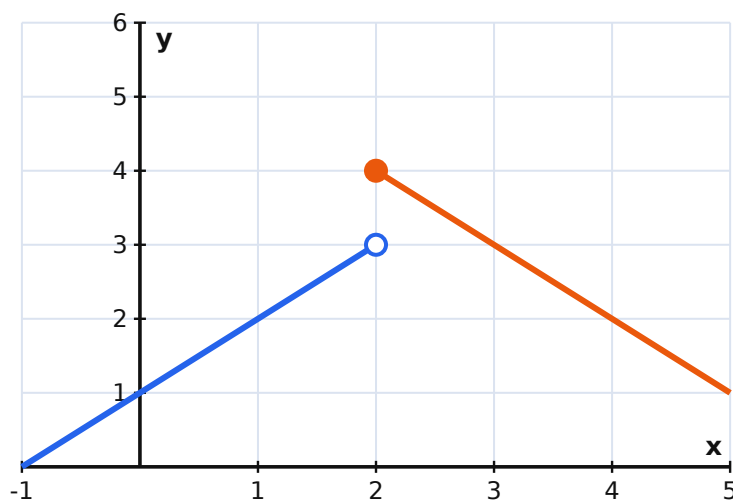
Example: $\lim_{x \rightarrow 3} \frac{x^2-9}{x-3} = \lim_{x \rightarrow 3} (x+3) = 6$. Another: $\lim_{x \rightarrow \infty} \frac{2x+1}{x-1} = 2$ because the leading coefficients are 2 and 1.



As $x \rightarrow \pm\infty$, $f(x) \rightarrow 2$: the horizontal asymptote is the limit at infinity.

What are one-sided limits and continuity?

The left-hand limit $\lim_{x \rightarrow a^-} f(x)$ uses only $x < a$, and the right-hand limit $\lim_{x \rightarrow a^+} f(x)$ uses only $x > a$. The two-sided limit exists exactly when both one-sided limits exist and are equal. A function is **continuous** at a if $f(a)$ exists, the limit exists, and the two are equal.



A jump: the left limit is 3 and the right limit is 4, so the limit does not exist at $x = 2$.

Discontinuities come in three kinds: a hole (removable), a jump (the one-sided limits differ) and a vertical asymptote (infinite limit).

Questions about introduction to limits

What does a limit tell you?

It tells the value a function is heading toward as the input approaches a number, regardless of

the value at that number. It is the foundation of derivatives and integrals.

Why does $0/0$ not mean the limit does not exist?

A $0/0$ form is indeterminate: the limit may or may not exist. Factor or rationalize to cancel the problem factor, then try substitution again.

What is a one-sided limit?

It is the value approached from one direction only: from the left (x less than a) or from the right (x greater than a). The two-sided limit exists only if both are equal.

How do you find a limit at infinity of a rational function?

Compare degrees. If the numerator's degree is lower, the limit is 0. If degrees are equal, divide the leading coefficients. If the numerator's degree is higher, the limit is infinite.

What is the difference between a limit and a function value?

The function value $f(a)$ is what the function equals at a . The limit is what it approaches near a . They can be different, as at a hole or a jump.