

Precalculus - Introduction to Limits - Answer Key

Common Core F-IF.B.6, F-IF.C.7d · maths-pdf.fr/en/

Use this key to check each limit. Solutions show the table, the algebraic simplification or the degree comparison, with final answers in bold.

Precalculus · Common Core State Standards: **F-IF.B.6, F-IF.C.7d**

Exercise 1 - Average rate of change

$$\frac{f(3)-f(1)}{3-1} = \frac{9-1}{2}.$$

4

Exercise 2 - Average rate of a cubic

$$\frac{8-1}{1}.$$

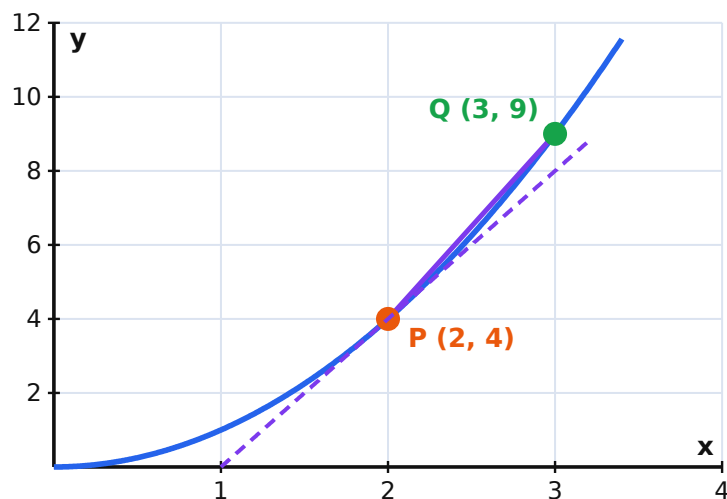
7

Exercise 3 - Average velocity

$$\frac{s(3)-s(1)}{2} = \frac{144-16}{2}.$$

64 ft/s

Exercise 4 - Shrinking interval



The secant slope approaches the tangent slope 4.

$$\frac{(2+h)^2-4}{h} = \frac{4h+h^2}{h} = 4+h. \text{ As } h \rightarrow 0 \text{ this approaches } 4.$$

$4+h$; the slope approaches 4

Exercise 5 - Limit from a table

x	2.9	2.99	3.01	3.1
f(x)	5.9	5.99	6.01	6.1

The values approach 6.

For $x \neq 3$, $f(x) = x+3$. The values approach 6 from both sides.

The limit is 6

Exercise 6 - Direct substitution

The function is a polynomial, so substitute: $12-2+1$.

11

Exercise 7 - Factor and cancel

$$\frac{(x-1)(x+1)}{x+1} = x-1 \text{ for } x \neq -1. \text{ The limit is } -1-1.$$

-2

Exercise 8 - Rationalize

Multiply by $\frac{\sqrt{x}+2}{\sqrt{x}+2}$: $\frac{x-4}{(x-4)(\sqrt{x}+2)} = \frac{1}{\sqrt{x}+2}$. At $x=4$: $\frac{1}{4}$.

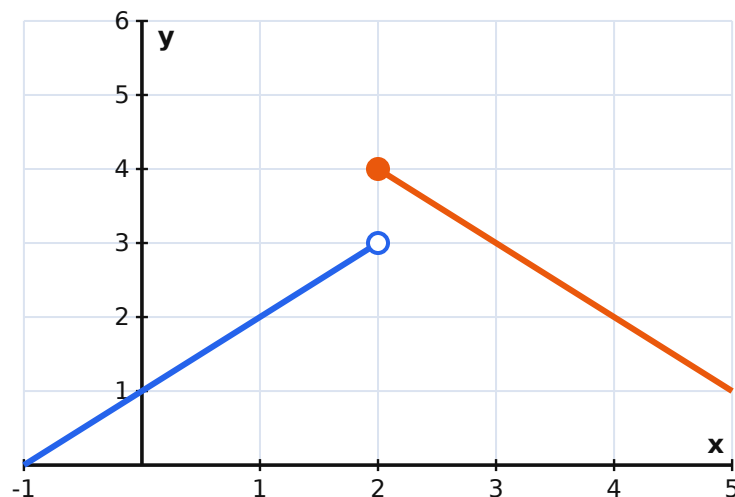
$\frac{1}{4}$

Exercise 9 - Expand first

$(x+5)^2 - 25 = x^2 + 10x$, so the quotient is $x+10$ for $x \neq 0$.

10

Exercise 10 - One-sided limits from a graph



Left limit 3; right limit 4; $f(2) = 4$.

The left piece heads to height 3 (open circle). The right piece starts at height 4 (closed point). The one-sided limits differ.

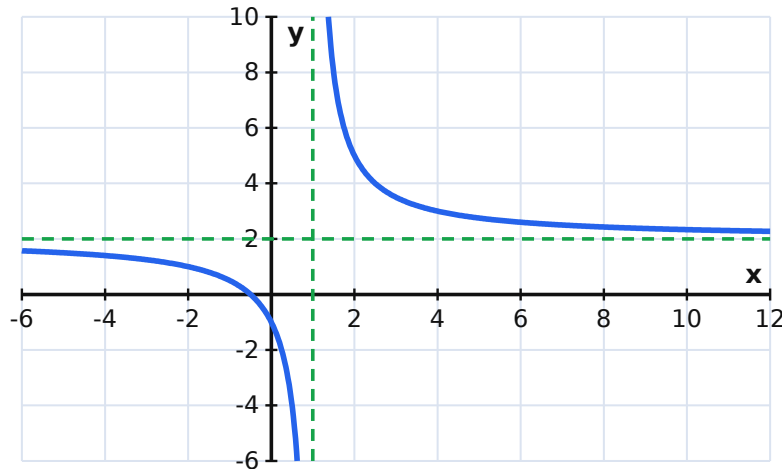
3, 4, $f(2) = 4$; **the limit does not exist**

Exercise 11 - A piecewise limit

Left: $2(3) - 1 = 5$. Right: $3 + 2 = 5$. $f(3) = 5$.

Both limits are 5 and $f(3) = 5$, so it is continuous

Exercise 12 - Limit at infinity (equal degrees)



The graph levels off at $y = 2$.

Divide numerator and denominator by x : $\frac{2+\frac{1}{x}}{1-\frac{1}{x}} \rightarrow \frac{2}{1}$.

2

Exercise 13 - Limit at infinity (higher powers)

Divide by x^2 : $\frac{3-\frac{4}{x^2}}{1+\frac{2}{x^2}} \rightarrow 3$.

3

Exercise 14 - Limit at infinity (lower numerator)

The denominator grows faster. Dividing by x^2 : $\frac{\frac{4}{x}+\frac{1}{x^2}}{1+\frac{2}{x^2}} \rightarrow 0$.

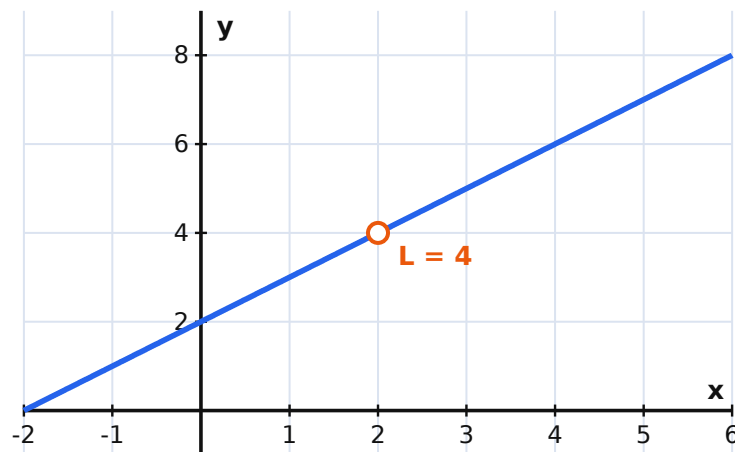
0

Exercise 15 - Infinite one-sided limits

For x slightly above 2, $x-2$ is a tiny positive number, so the quotient is huge and positive. Slightly below 2 it is huge and negative.

∞ and $-\infty$; the two-sided limit does not exist

Exercise 16 - Removable discontinuity



The point (2, 5) sits above the hole at (2, 4).

$\lim_{x \rightarrow 2} f(x) = 4$ but $f(2) = 5$, so it is not continuous. Redefine $f(2) = 4$ to fill the hole.

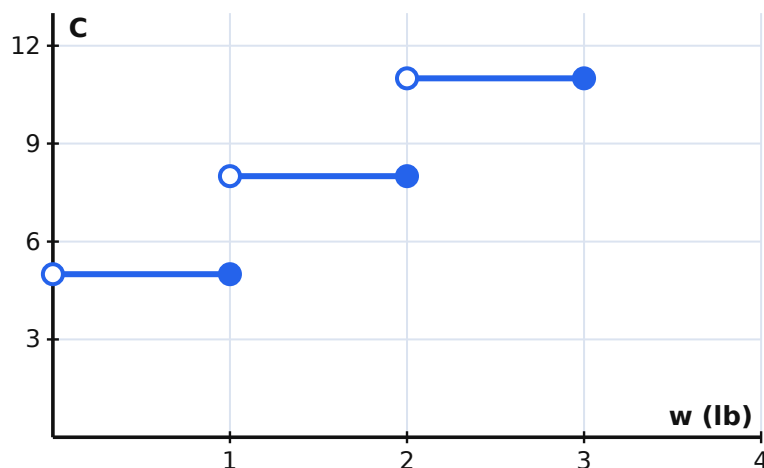
Not continuous; set $f(2) = 4$

Exercise 17 - Make a function continuous

Left limit: $4+1=5$. Right: $2k-1$. Set equal: $2k-1=5$.

$$k = 3$$

Exercise 18 - Delivery fee



Closed circle at the right end of each step.

Just below 1 lb the fee is \$5; just above it is \$8. The limit does not exist at $w=1$.

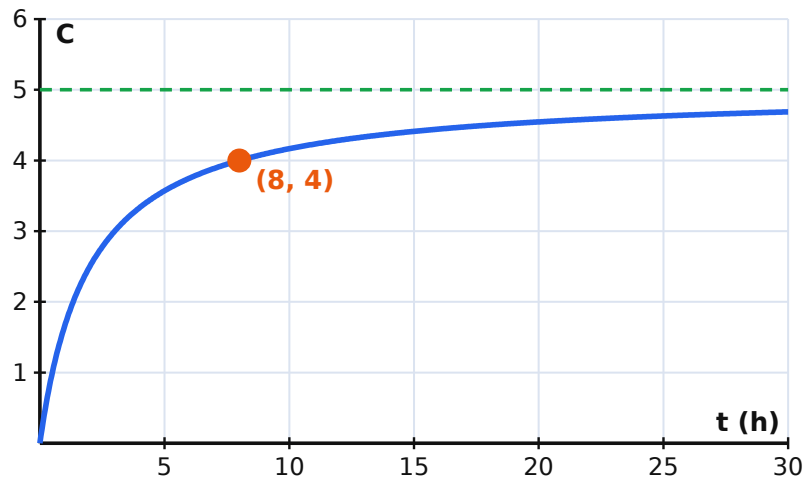
Left \$5; right \$8; no two-sided limit

Exercise 19 - Instantaneous velocity

$\frac{(3+h)^2 + 2(3+h) - 15}{h} = \frac{8h + h^2}{h} = 8 + h$. As $h \rightarrow 0$ the limit is 8. Check: $h=0.1$ gives 8.1; $h=0.01$ gives 8.01.

$8+h$; 8 ft/s

Exercise 20 - Drug concentration



The curve approaches 5 mg/L.

$C(8) = \frac{40}{10} = 4$. Equal degrees: the limit is $\frac{5}{1} = 5$. The concentration approaches 5 mg/L but never exceeds it.

4 mg/L; 5 mg/L