

Precalculus - Introduction to Limits - Worksheet

Common Core F-IF.B.6, F-IF.C.7d · maths-pdf.fr/en/

Name: _____ Class: _____ Date: ____/____/____

These 20 problems introduce limits (Common Core F-IF.B.6 and F-IF.C.7d). You will find average rates of change, estimate limits from tables and graphs, evaluate limits algebraically, use one-sided limits and limits at infinity and test continuity.

Precalculus · Common Core State Standards: **F-IF.B.6, F-IF.C.7d**

Exercise 1 - Average rate of change

Find the average rate of change of $f(x) = x^2$ from $x = 1$ to $x = 3$.

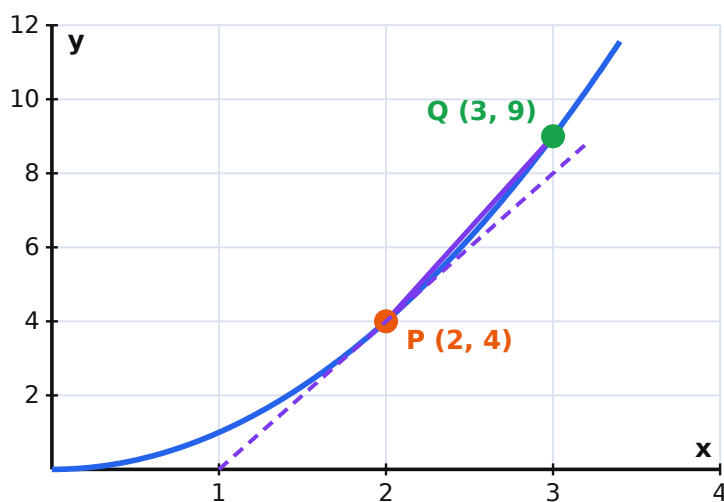
Exercise 2 - Average rate of a cubic

Find the average rate of change of $f(x) = x^3$ from $x = 1$ to $x = 2$.

Exercise 3 - Average velocity

A rock falls so that $s(t) = 16t^2$ feet after t seconds. Find the average velocity from $t = 1$ to $t = 3$.

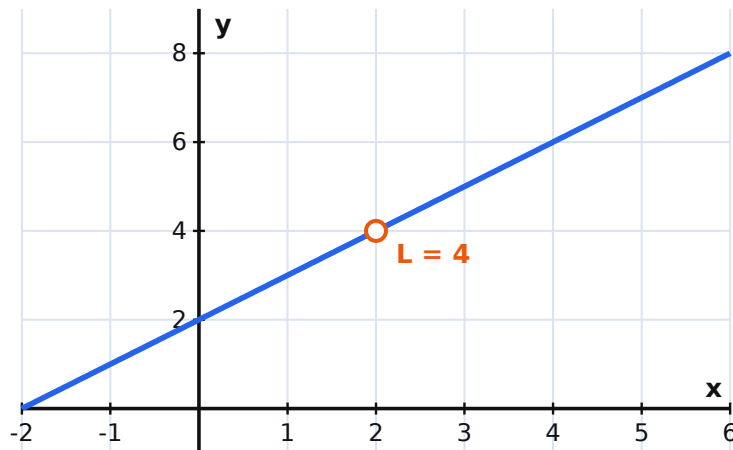
Exercise 4 - Shrinking interval



Secant lines from P.

For $f(x) = x^2$, find the average rate of change on $[2, 2+h]$ and describe what happens as $h \rightarrow 0$.

Exercise 5 - Limit from a table



A graph with a hole.

Complete the table for $f(x) = \frac{x^2-9}{x-3}$ at $x = 2.9, 2.99, 3.01, 3.1$ and estimate $\lim_{x \rightarrow 3} f(x)$.

Exercise 6 - Direct substitution

Evaluate $\lim_{x \rightarrow 2} (3x^2 - x + 1)$.

Exercise 7 - Factor and cancel

Evaluate $\lim_{x \rightarrow -1} \frac{x^2-1}{x+1}$.

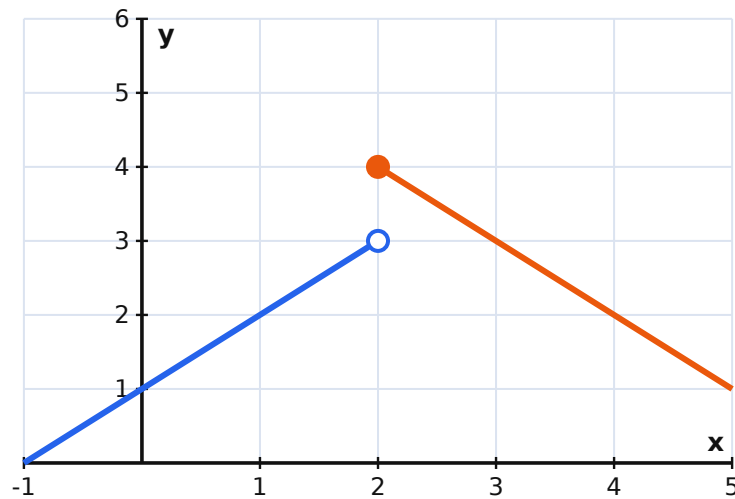
Exercise 8 - Rationalize

Evaluate $\lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x-4}$.

Exercise 9 - Expand first

Evaluate $\lim_{x \rightarrow 0} \frac{(x+5)^2-25}{x}$.

Exercise 10 - One-sided limits from a graph



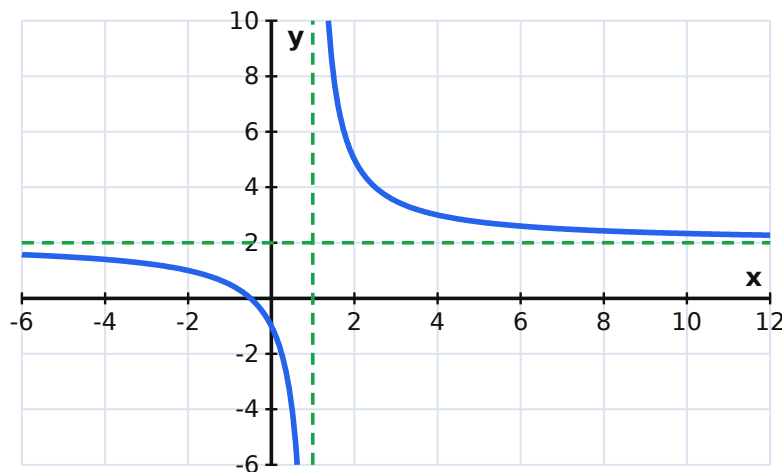
A graph with a jump at $x = 2$.

Use the graph to find $\lim_{x \rightarrow 2^-} f(x)$, $\lim_{x \rightarrow 2^+} f(x)$, $f(2)$ and say whether $\lim_{x \rightarrow 2} f(x)$ exists.

Exercise 11 - A piecewise limit

Let $f(x) = 2x - 1$ for $x < 3$ and $f(x) = x + 2$ for $x \geq 3$. Find the one-sided limits at $x = 3$ and decide if f is continuous at 3.

Exercise 12 - Limit at infinity (equal degrees)



Graph with a horizontal asymptote.

Evaluate $\lim_{x \rightarrow \infty} \frac{2x+1}{x-1}$.

Exercise 13 - Limit at infinity (higher powers)

Evaluate $\lim_{x \rightarrow \infty} \frac{3x^2-5x}{x^2+4}$.

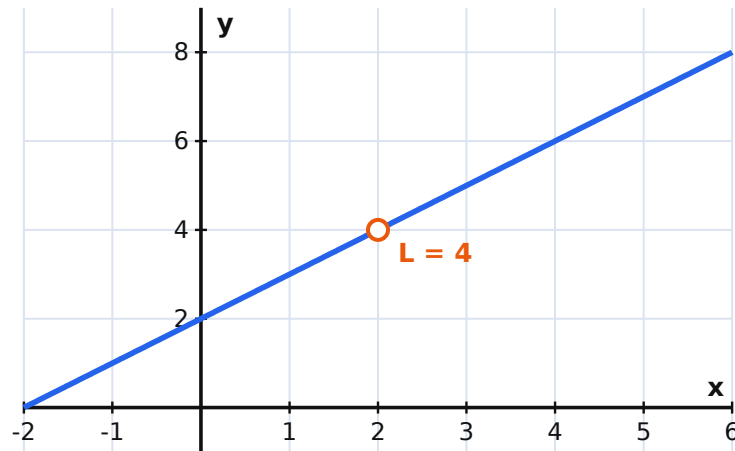
Exercise 14 - Limit at infinity (lower numerator)

Evaluate $\lim_{x \rightarrow \infty} \frac{4x+1}{x^2+2}$.

Exercise 15 - Infinite one-sided limits

Describe $\lim_{x \rightarrow 2^+} \frac{1}{x-2}$ and $\lim_{x \rightarrow 2^-} \frac{1}{x-2}$.

Exercise 16 - Removable discontinuity



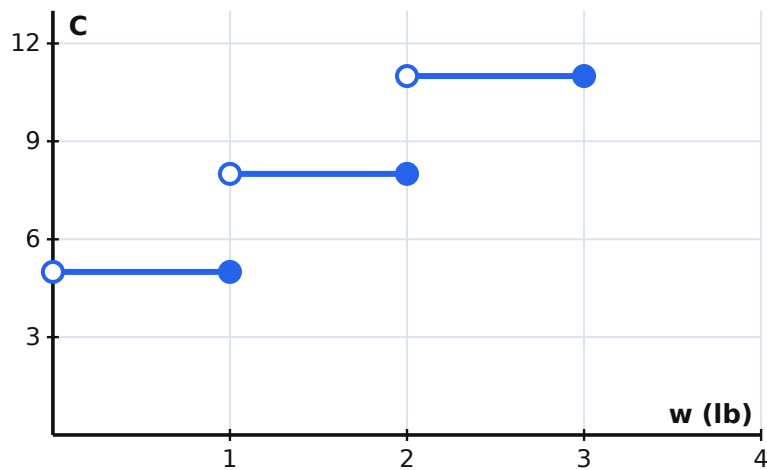
Graph of f.

Let $f(x) = \frac{x^2-4}{x-2}$ for $x \neq 2$ and $f(2) = 5$. Is f continuous at 2? How can you remove the discontinuity?

Exercise 17 - Make a function continuous

Find k so that $f(x) = x^2+1$ for $x < 2$ and $f(x) = kx-1$ for $x \geq 2$ is continuous at 2.

Exercise 18 - Delivery fee



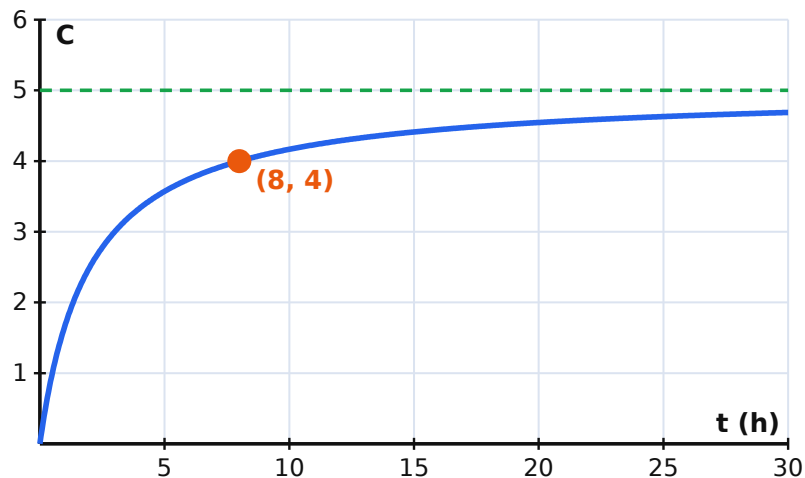
Delivery fee by weight.

A store charges \$5 to ship up to 1 lb, \$8 for over 1 lb up to 2 lb, and \$11 for over 2 lb up to 3 lb. Find the left and right limits of the fee $C(w)$ at $w=1$.

Exercise 19 - Instantaneous velocity

A particle's position is $s(t) = t^2 + 2t$ feet. Find the average velocity on $[3, 3+h]$ and the instantaneous velocity at $t=3$.

Exercise 20 - Drug concentration



Concentration over time.

The concentration of a drug is $C(t) = \frac{5t}{t+2}$ mg/L after t hours. Find $C(8)$ and $\lim_{t \rightarrow \infty} C(t)$, and explain what the limit means.