

Precalculus - Logarithms and Their Properties - Lesson

Common Core F-LE.A.4, F-IF.C.7e, F-BF.B.5 · maths-pdf.fr/en/

A logarithm is an exponent: $\log_b x = y$ means $b^y = x$. Logarithms and exponentials are inverse functions, so $\log_b x$ is defined only for $x > 0$ and its graph is the reflection of b^x over $y = x$. The product, quotient and power rules turn multiplication into addition, division into subtraction, and powers into multiplication.

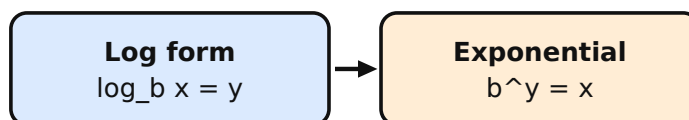
Precalculus · Common Core State Standards: **F-LE.A.4, F-IF.C.7e, F-BF.B.5**

Key facts

- $\log_b x = y$ if and only if $b^y = x$ ($b > 0$, $b \neq 1$, $x > 0$).
- $\log_b 1 = 0$ and $\log_b b = 1$.
- Product rule adds logs, quotient rule subtracts, power rule multiplies.
- Change of base: $\log_b M = \ln M / \ln b$.
- The graph of $\log_b x$ has the vertical asymptote $x = 0$ and passes through $(1, 0)$.

What is a logarithm?

A logarithm answers the question: what exponent is needed? We define $\log_b x = y$ to mean $b^y = x$, where $b > 0$, $b \neq 1$ and $x > 0$. For example $\log_2 32 = 5$ because $2^5 = 32$. The base-10 logarithm is written $\log x$ and the base- e (natural) logarithm is written $\ln x$.

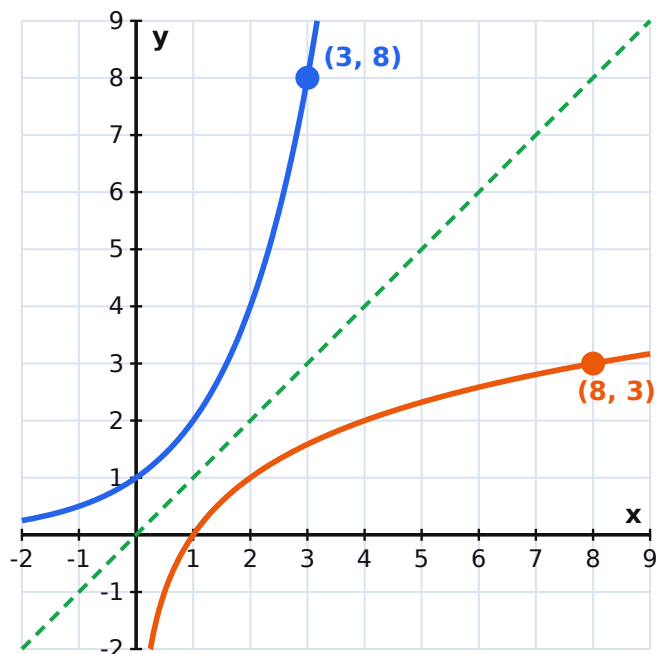


A logarithm is an exponent.

Two values come straight from the definition: $\log_b 1 = 0$ and $\log_b b = 1$. Also $b^{\log_b x} = x$ and $\log_b b^x = x$, because the logarithm and the exponential undo each other.

How is a logarithm graph related to an exponential graph?

$f(x) = b^x$ and $g(x) = \log_b x$ are inverse functions, so their graphs are mirror images across $y = x$. The log graph has domain $(0, \infty)$, range all real numbers, a vertical asymptote $x = 0$ and the point $(1, 0)$.

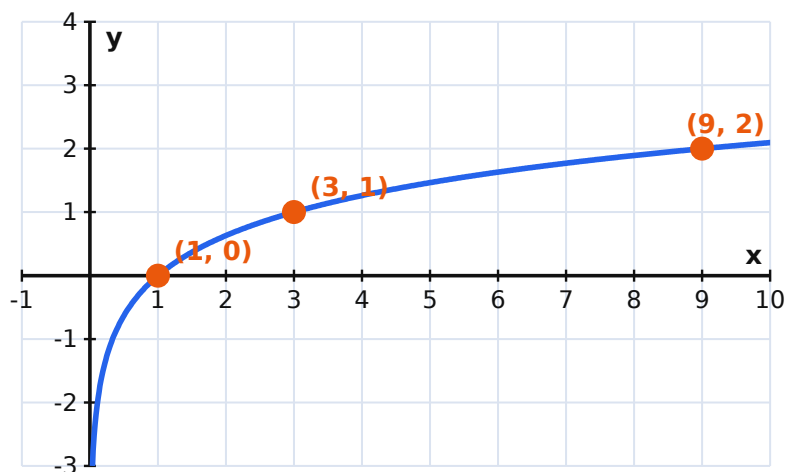


The point $(3, 8)$ on 2^x matches $(8, 3)$ on $\log_2 x$.

What are the properties of logarithms?

Rule	Formula
Product	$\log_b(MN) = \log_b M + \log_b N$
Quotient	$\log_b \frac{M}{N} = \log_b M - \log_b N$
Power	$\log_b M^p = p \log_b M$
Change of base	$\log_b M = \frac{\ln M}{\ln b}$

To **expand** a logarithm, apply the rules to split it into simple pieces. To **condense** it, work backward. **Example:** $\log_2 \frac{8x^3}{y} = \log_2 8 + 3\log_2 x - \log_2 y = 3 + 3\log_2 x - \log_2 y$.

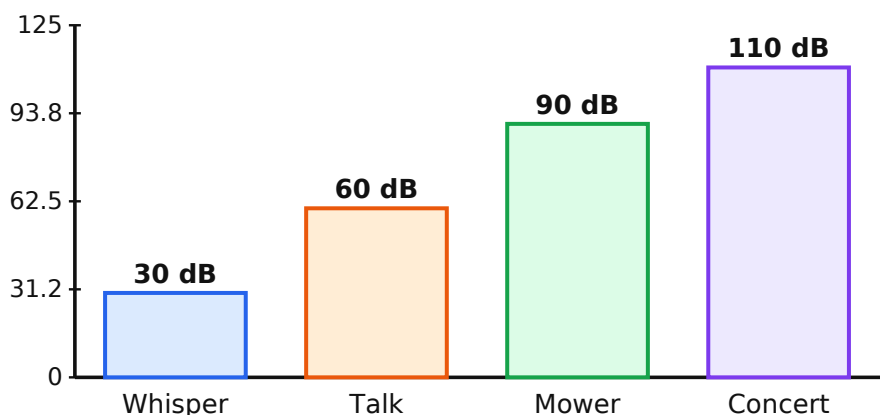


$y = \log_3 x$ passes through (1,0), (3,1) and (9,2): each factor of 3 adds 1.

Example (change of base): Most calculators have only $\log$ and $\ln$ keys. To find $\log_3 20$, compute $\frac{\ln 20}{\ln 3} \approx \frac{2.9957}{1.0986} \approx 2.727$. This makes sense because $3^2 = 9$ and $3^3 = 27$, and 20 lies between them, so the exponent must be between 2 and 3. Estimating between two known powers is a quick way to catch a calculator mistake. You can also check your answer by raising the base to it: $3^{2.727} \approx 20$.

Where are logarithms used?

Logarithms measure quantities that span huge ranges. Sound level in decibels is $L = 10 \log \frac{I}{I_0}$, acidity is $\text{pH} = -\log[\text{H}^+]$, and earthquake magnitude grows with the logarithm of the wave amplitude. Each step of 1 on a log scale means a factor of 10 in the quantity, so a 20 dB increase is a 100 times larger intensity.



Decibel levels: each 10 dB is ten times the intensity.

Remember that only positive numbers have logarithms. The expression $\log(x-3)$ is defined only for $x > 3$, and a graph shifted by h has its vertical asymptote at $x = h$.

Questions about logarithms and their properties

What does log base 2 of 8 mean?

It asks which power of 2 gives 8. Since 2 cubed is 8, log base 2 of 8 equals 3.

What is the difference between log and ln?

The notation $\log$ usually means base 10, and $\ln$ means base e , about 2.718. Both follow the same properties.

Why can't you take the log of a negative number?

A positive base raised to any real power is always positive, so no exponent produces a negative number or zero. The domain of a logarithm is $x > 0$.

How do you evaluate a logarithm with a calculator?

Use the change of base formula. For log base 3 of 20, compute $\ln 20$ divided by $\ln 3$, which is about 2.727.

Does $\log(A + B)$ equal $\log A + \log B$?

No. The product rule applies to $\log(AB)$, not $\log(A + B)$. For example $\log(10 + 10) = \log 20$, not 2.