

Precalculus - Logarithms and Their Properties - Answer Key

Common Core F-LE.A.4, F-IF.C.7e, F-BF.B.5 · maths-pdf.fr/en/

Use this key to check each logarithm. Every solution shows the conversion to exponential form or the rule used, and final answers are in bold.

Precalculus · Common Core State Standards: **F-LE.A.4, F-IF.C.7e, F-BF.B.5**

Exercise 1 - Exponential to log form

The base is 2, the exponent is 5 and the result is 32.

$$\log_2 32 = 5$$

Exercise 2 - Log to exponential form

The base is the base; the log is the exponent.

$$3^4 = 81 \text{ and } 5^{-2} = \frac{1}{25}$$

Exercise 3 - Evaluate a log

Ask: $4^? = 64$. $4^3 = 64$.

3

Exercise 4 - Log of a fraction

$$2^{-3} = \frac{1}{8}$$

-3

Exercise 5 - Common and natural logs

$10^3 = 1000$. $\ln e^5 = 5$ because $\ln$ undoes e^x .

Exercise 6 - A fractional answer

$$9^{1/2} = 3.$$

$$\frac{1}{2}$$

Exercise 7 - Domain

Require $x-3 > 0$.

Domain $(3, \infty)$; **asymptote** $x = 3$

Exercise 8 - Expand

Quotient rule: $\log_2(8x^3) - \log_2 y$. Product and power rules: $\log_2 8 + 3\log_2 x - \log_2 y$. Since $\log_2 8 = 3$:
 $3 + 3\log_2 x - \log_2 y$

Exercise 9 - Condense

Power rule: $\log x^2 + \log y - \log z^3$. Then product and quotient rules.

$$\log \frac{x^2 y}{z^3}$$

Exercise 10 - Use given values

$$\log_b 6 = 0.30 + 0.48 = 0.78, \log_b 1.5 = \log_b \frac{3}{2} = 0.48 - 0.30 = 0.18, \log_b 8 = 3(0.30) = 0.90.$$

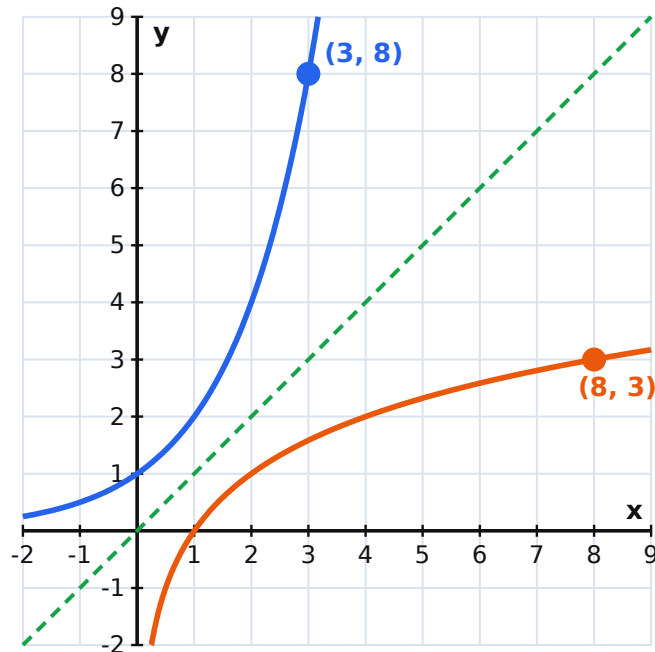
0.78, 0.18, 0.90

Exercise 11 - Change of base

$$\frac{\ln 20}{\ln 3} = \frac{2.9957}{1.0986} \approx 2.727.$$

2.727

Exercise 12 - Inverse functions

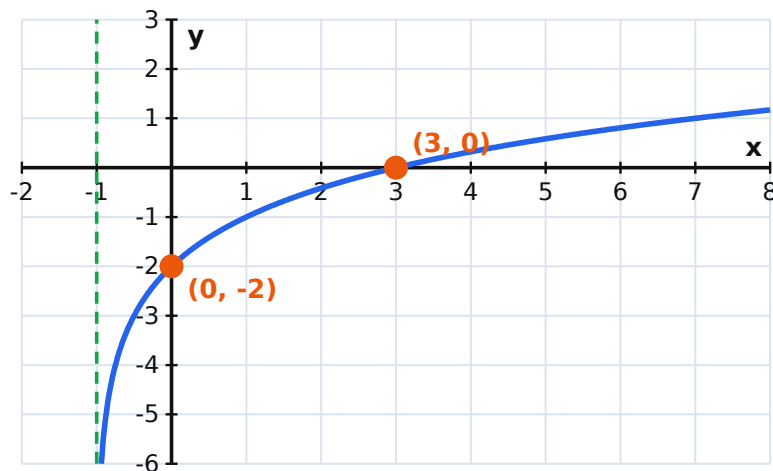


Swap the coordinates: $(8, 3)$.

The inverse swaps coordinates: $(8,3)$. Check: $\log_2 8 = 3$.

$(8,3)$

Exercise 13 - Transformed log graph



Asymptote $x = -1$; intercepts $(3, 0)$ and $(0, -2)$.

The graph is $\log_2 x$ shifted left 1 and down 2. Asymptote $x = -1$. $y = 0 : \log_2(x+1) = 2$, so

$$x+1=4, x=3. y\text{-intercept: } \log_2 1-2=-2.$$

$$x=-1; (3,0); (0,-2)$$

Exercise 14 - Inverse properties

Exponentials and logs undo each other, and $\ln 1 = 0$.

$$7, 4, 0, 3x$$

Exercise 15 - Powers inside logs

$$\log_2 16^3 = 3 \cdot 4 = 12. \log 0.01 = \log 10^{-2} = -2. \left(\frac{1}{2}\right)^{-3} = 8.$$

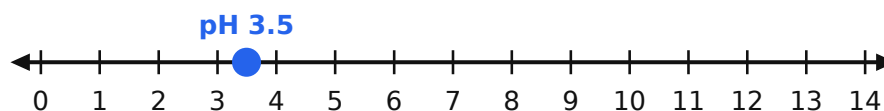
$$12, -2, -3$$

Exercise 16 - Decibels

The difference is 50 dB, so the ratio is $10^{50/10} = 10^5$.

100,000 **times as intense**

Exercise 17 - pH



pH of about 3.5.

$$-\log(3.2 \times 10^{-4}) \approx 3.49. \text{ For pH } 8: [\text{H}^+] = 10^{-8}.$$

$$\text{pH} \approx 3.5; 10^{-8}$$

Exercise 18 - Magnitude difference

$$7-5=2, \text{ so } 10^2.$$

100 **times**

Exercise 19 - Verify a property

$$12 = 4 \cdot 3, \text{ so } \log_2 12 = \log_2 4 + \log_2 3 = 2 + \log_2 3 \approx 2 + 1.585.$$

$$\approx 3.585$$

Exercise 20 - Solve for the base or argument

$$(a) x^4 = 81, x = 3. (b) 2^{-4} = \frac{1}{16}. (c) 5^3 = 125.$$

$$(a) 3; (b) \frac{1}{16}; (c) 3$$