

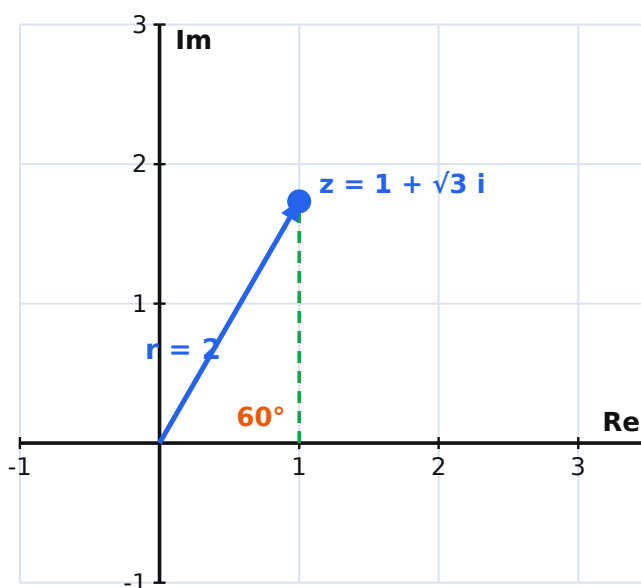
Precalculus - Polar Form and De Moivre's Theorem - Answer Key

Common Core N-CN.B.4, N-CN.B.5, N-CN.B.6 · maths-pdf.fr/en/

Use this key to check each polar computation. Solutions show the modulus, the argument and the rule used, with final answers in bold.

Precalculus · Common Core State Standards: **N-CN.B.4, N-CN.B.5, N-CN.B.6**

Exercise 1 - Modulus and argument



$r = 2$ and angle 60 degrees.

$$r = \sqrt{1+3} = 2. \tan \theta = \sqrt{3} \text{ in quadrant I, so } \theta = 60^\circ.$$

$$z = 2(\cos 60^\circ + i \sin 60^\circ)$$

Exercise 2 - Quadrant II

$$r = \sqrt{9+9} = 3\sqrt{2}. \text{ The point is in quadrant II with reference angle } 45^\circ, \text{ so } \theta = 135^\circ.$$

$$3\sqrt{2}(\cos 135^\circ + i \sin 135^\circ)$$

Exercise 3 - On an axis

$r = 4$. The point $(0, -4)$ is on the negative imaginary axis, at 270° .

$$4(\cos 270^\circ + i \sin 270^\circ)$$

Exercise 4 - Polar to rectangular

$$6 \cos 120^\circ = -3, \quad 6 \sin 120^\circ = 3\sqrt{3}.$$

$$-3 + 3\sqrt{3}i$$

Exercise 5 - Polar to rectangular in radians

$$4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}, \quad 4 \cdot \frac{1}{2} = 2.$$

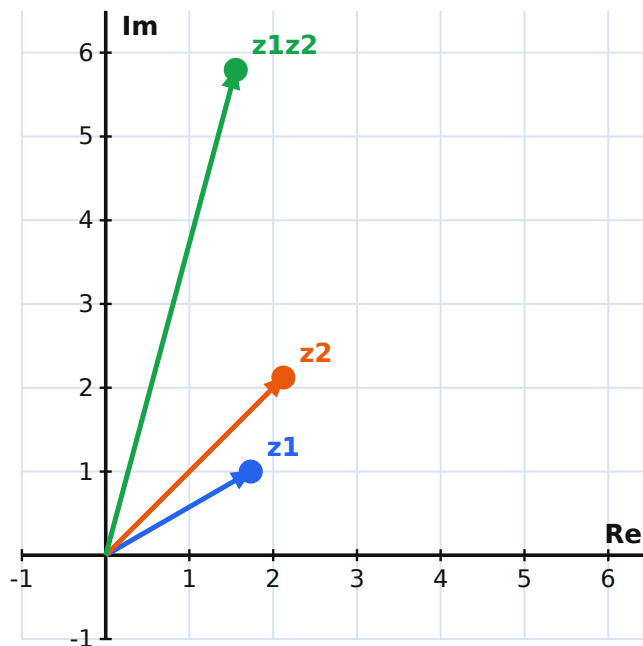
$$2\sqrt{3} + 2i$$

Exercise 6 - Quadrant IV

$r = 5\sqrt{2}$. The point is in quadrant IV with reference angle 45° , so $\theta = 315^\circ$ (or -45°).

$$5\sqrt{2}(\cos 315^\circ + i \sin 315^\circ)$$

Exercise 7 - Multiply in polar form



$6 \text{ cis } 75^\circ$.

Moduli: $2 \cdot 3 = 6$. Angles: $30^\circ + 45^\circ = 75^\circ$. Rectangular: $6 \cos 75^\circ \approx 1.553$, $6 \sin 75^\circ \approx 5.796$.

$$6(\cos 75^\circ + i \sin 75^\circ) \approx 1.553 + 5.796i$$

Exercise 8 - Divide in polar form

$\frac{12}{4} = 3$ and $200^\circ - 50^\circ = 150^\circ$. Rectangular: $3 \cos 150^\circ \approx -2.598$ and $3 \sin 150^\circ = 1.5$.

$$3 \text{ cis } 150^\circ \approx -2.598 + 1.500i$$

Exercise 9 - Check with rectangular multiplication

$1+i = \sqrt{2} \text{ cis } 45^\circ$ and $\sqrt{3}+i = 2 \text{ cis } 30^\circ$. Product: $2\sqrt{2} \text{ cis } 75^\circ \approx 0.732 + 2.732i$. Rectangular: $\sqrt{3}+i + \sqrt{3}i + i^2 = (\sqrt{3}-1) + (\sqrt{3}+1)i \approx 0.732 + 2.732i$.

$$2\sqrt{2} \text{ cis } 75^\circ \approx 0.732 + 2.732i$$

Exercise 10 - De Moivre with a unit modulus

$$\cos(9 \cdot 20^\circ) + i \sin(9 \cdot 20^\circ) = \cos 180^\circ + i \sin 180^\circ.$$

$$-1$$

Exercise 11 - Power of $1 + i$

$$1+i = \sqrt{2} \operatorname{cis} 45^\circ. (\sqrt{2})^8 = 16 \text{ and } 8 \cdot 45^\circ = 360^\circ.$$

16

Exercise 12 - Power with a negative angle

$$r=2, \theta=-30^\circ. 2^6=64 \text{ and } 6 \cdot (-30^\circ) = -180^\circ: 64 \operatorname{cis}(-180^\circ).$$

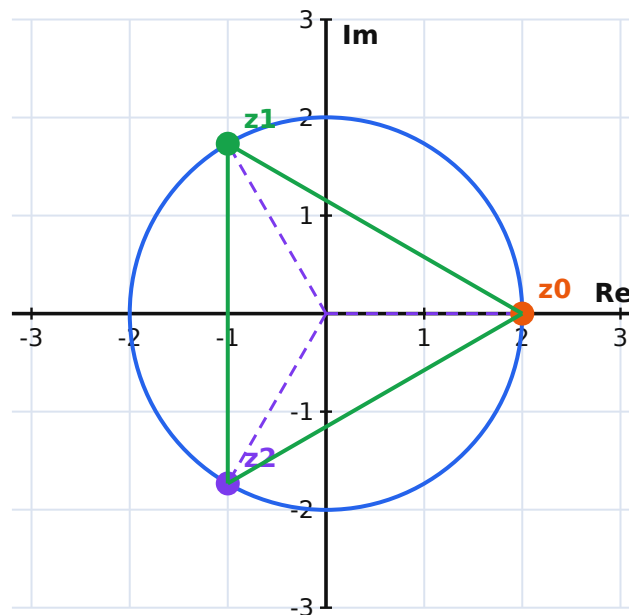
$$-64$$

Exercise 13 - Power in rectangular form

$$2^4=16, 4 \cdot 15^\circ = 60^\circ: 16 \cdot \frac{1}{2} + 16 \cdot \frac{\sqrt{3}}{2} i.$$

$$8 + 8\sqrt{3} i$$

Exercise 14 - Cube roots of 8

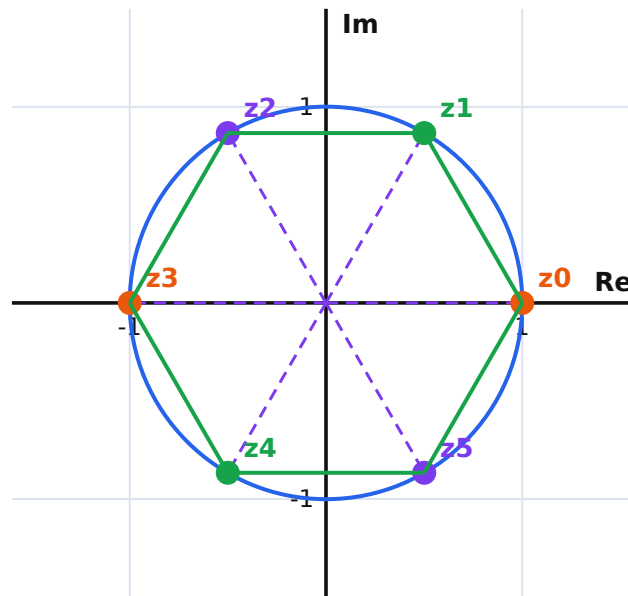


z_0, z_1, z_2 at $0, 120$ and 240 degrees.

$$8 = 8 \operatorname{cis} 0^\circ. \text{ The roots have modulus } \sqrt[3]{8} = 2 \text{ and arguments } 0^\circ, 120^\circ, 240^\circ: 2, -1 + \sqrt{3} i, -1 - \sqrt{3} i.$$

$$2, -1 + \sqrt{3} i, -1 - \sqrt{3} i$$

Exercise 15 - Sixth roots of unity



Six points spaced 60 degrees apart.

Since $1 = 1 \operatorname{cis} 0^\circ$, $\theta = \frac{360^\circ k}{6} = 60^\circ k$ for $k = 0$, to 5 .

$\operatorname{cis} 0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ$

Exercise 16 - Fourth roots of -16

$-16 = 16 \operatorname{cis} 180^\circ$. Modulus $\sqrt[4]{16} = 2$. Arguments $\frac{180^\circ + 360^\circ k}{4} = 45^\circ, 135^\circ, 225^\circ, 315^\circ$. Each has $2 \cos 45^\circ = \sqrt{2}$.

$\sqrt{2} + \sqrt{2}i, -\sqrt{2} + \sqrt{2}i, -\sqrt{2} - \sqrt{2}i, \sqrt{2} - \sqrt{2}i$

Exercise 17 - Solve $z^3 = 27i$

$27i = 27 \operatorname{cis} 90^\circ$. Roots: $3 \operatorname{cis} 30^\circ, 3 \operatorname{cis} 150^\circ, 3 \operatorname{cis} 270^\circ$. That is $\frac{3\sqrt{3}}{2} + \frac{3}{2}i, -\frac{3\sqrt{3}}{2} + \frac{3}{2}i$ and $-3i$.

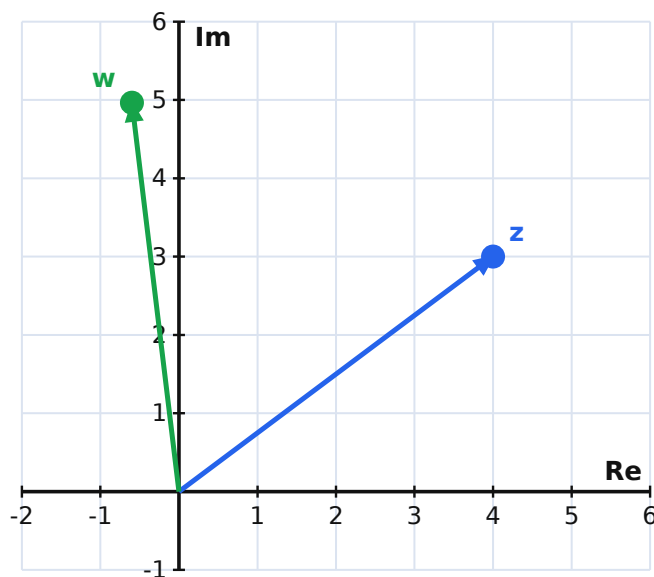
$z = \frac{3\sqrt{3}}{2} + \frac{3}{2}i, -\frac{3\sqrt{3}}{2} + \frac{3}{2}i, -3i$

Exercise 18 - AC circuit

$|Z| = 5$ and $\arg Z = \arctan \frac{4}{3} \approx 53.13^\circ$. Multiply: $10 \cdot 5 = 50, 0^\circ + 53.13^\circ$.

$$Z = 5 \operatorname{cis} 53.13^\circ; V = 50 \operatorname{cis} 53.13^\circ \text{ volts}$$

Exercise 19 - Rotate a point



The point moves along a circle of radius 5.

$$(4+3i)\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) = 2+2\sqrt{3}i+\frac{3}{2}i+\frac{3\sqrt{3}}{2}i^2 = \left(2-\frac{3\sqrt{3}}{2}\right) + \left(2\sqrt{3}+\frac{3}{2}\right)i \approx -0.60+4.96i.$$

$$\approx -0.60+4.96i$$

Exercise 20 - Cube root of unity

$$\omega = \operatorname{cis} 120^\circ, \text{ so } \omega^3 = \operatorname{cis} 360^\circ = 1. \omega^2 = \operatorname{cis} 240^\circ = -\frac{1}{2}-\frac{\sqrt{3}}{2}i. \text{ Then } 1+\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)+\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=0.$$

Both statements hold