

Precalculus - Polar Form and De Moivre's Theorem - Test Answer Key

Common Core N-CN.B.4, N-CN.B.5, N-CN.B.6 · maths-pdf.fr/en/

Worked solutions and scoring notes for the 9-question polar form test. Use the scoring line under each solution to award partial credit.

Precalculus · Common Core State Standards: **N-CN.B.4, N-CN.B.5, N-CN.B.6**

Question 1 - Polar form [2 pts]

$r = \sqrt{12+4} = 4$. Quadrant IV with reference angle $\arctan \frac{2}{2\sqrt{3}} = 30^\circ$, so $\theta = 330^\circ$.

$$4(\cos 330^\circ + i \sin 330^\circ)$$

Scoring: 1 point for r , 1 point for the angle.

Question 2 - Rectangular form [2 pts]

$$8\cos 225^\circ = -4\sqrt{2} \text{ and } 8\sin 225^\circ = -4\sqrt{2}.$$

$$-4\sqrt{2} - 4\sqrt{2}i$$

Scoring: 1 point for each part.

Question 3 - Product [2 pts]

$$10 \operatorname{cis} 120^\circ. \text{ Rectangular: } 10\cos 120^\circ = -5 \text{ and } 10\sin 120^\circ = 5\sqrt{3}.$$

$$10 \operatorname{cis} 120^\circ = -5 + 5\sqrt{3}i$$

Scoring: 1 point for polar form, 1 point for rectangular.

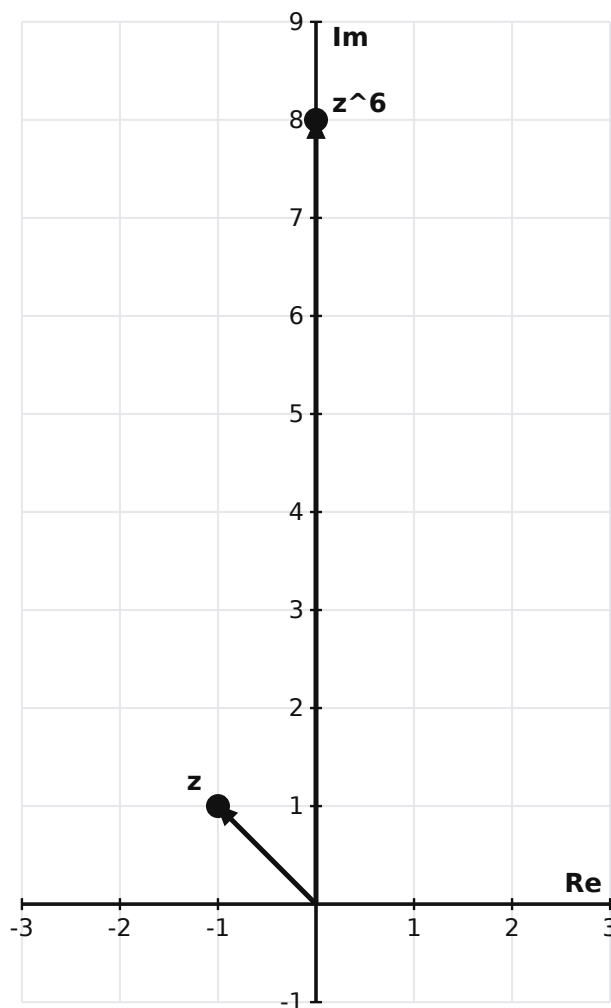
Question 4 - Quotient [2 pts]

$$\frac{18}{6} = 3 \text{ and } 280^\circ - 100^\circ = 180^\circ.$$

$$3 \operatorname{cis} 180^\circ = -3$$

Scoring: 1 point for the modulus, 1 point for the angle.

Question 5 - Power with De Moivre [3 pts]



z^6 lies on the positive imaginary axis.

(a) $r = \sqrt{2}$, $\theta = 135^\circ$. (b) $z^6 = (\sqrt{2})^6 \text{cis}(810^\circ) = 8 \text{cis} 90^\circ$. (c) $8i$.

$\sqrt{2} \text{cis} 135^\circ$; $8 \text{cis} 90^\circ$; $8i$

Scoring: 1 point for each part.

Question 6 - Large power [2 pts]

$1-i = \sqrt{2} \text{cis}(-45^\circ)$. $(\sqrt{2})^{10} = 32$, $10 \cdot (-45^\circ) = -450^\circ \equiv -90^\circ$.

$32 \text{cis}(-90^\circ) = -32i$

Scoring: 1 point for the modulus power, 1 point for the angle and result.

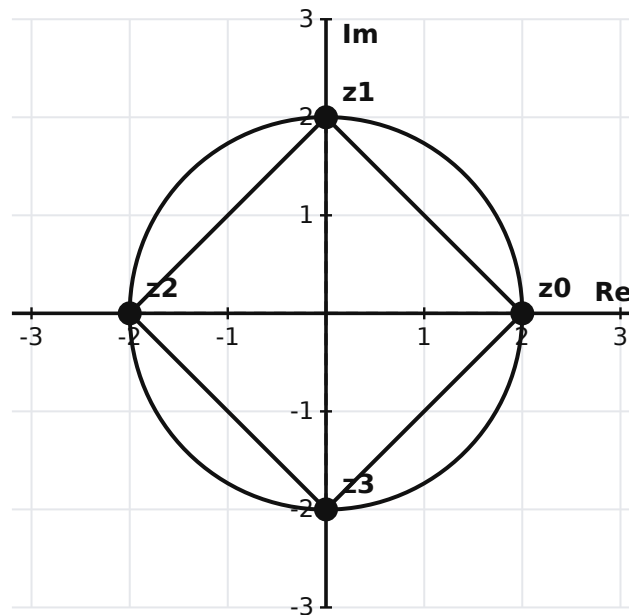
Question 7 - Cube roots [2 pts]

$$-27 = 27 \operatorname{cis} 180^\circ. \text{ Roots: } 3 \operatorname{cis} \frac{180^\circ + 360^\circ k}{3} \text{ for } k=0,1,2.$$

$$3 \operatorname{cis} 60^\circ, 3 \operatorname{cis} 180^\circ, 3 \operatorname{cis} 300^\circ$$

Scoring: 1 point for the modulus and method, 1 point for the three angles.

Question 8 - Fourth roots [3 pts]



Roots at 0, 90, 180 and 270 degrees.

(a) $16 \operatorname{cis} 0^\circ$. (b) Modulus 2, arguments $0^\circ, 90^\circ, 180^\circ, 270^\circ$: 2, $2i$, -2 , $-2i$. (c) They lie on one circle with equal angles of 90° between them.

$$16 \operatorname{cis} 0^\circ; 2, 2i, -2, -2i; \text{ equal spacing of } 90^\circ$$

Scoring: 1 point for each part.

Question 9 - Rotation [2 pts]

$$i(-2+5i) = -2i+5i^2.$$

$$-5-2i$$

Scoring: 1 point for the multiplication, 1 point for simplifying.

Total
20 points