

Precalculus - Polynomial Division and the Factor Theorem - Lesson

Common Core A-APR.B.2, A-APR.D.6, A-APR.B.3 · maths-pdf.fr/en/

To divide a polynomial $f(x)$ by $d(x)$, use long division, or synthetic division when $d(x) = x - c$. The Remainder Theorem says the remainder equals $f(c)$, and the Factor Theorem says $x - c$ is a factor exactly when $f(c) = 0$. The Rational Root Theorem lists the possible rational zeros p/q to test.

Precalculus · Common Core State Standards: **A-APR.B.2, A-APR.D.6, A-APR.B.3**

Key facts

- $f(x) = d(x)q(x) + r(x)$, with r of lower degree than d .
- Remainder Theorem: dividing by $x - c$ leaves $f(c)$.
- Factor Theorem: $x - c$ is a factor if and only if $f(c) = 0$.
- Rational Root Theorem: p divides the constant term and q divides the leading coefficient.
- Use 0 as a placeholder coefficient for each missing power.

How do you divide polynomials?

Polynomial long division works like long division of whole numbers. Divide the leading term of the dividend by the leading term of the divisor, multiply back, subtract, and bring down the next term. You get $f(x) = d(x)q(x) + r(x)$, where the remainder $r(x)$ has lower degree than the divisor $d(x)$. If the remainder is 0, the divisor is a factor.

What is synthetic division?

When the divisor has the form $x - c$, synthetic division is a fast shortcut that uses only the coefficients.

1. Write the coefficients of f in order, with 0 for any missing power, and put c to the left.
2. Bring down the first coefficient.
3. Multiply it by c , write the product under the next coefficient, and add.
4. Repeat. The last number is the remainder, and the others are the quotient coefficients.

Example: Divide $x^3 + 4x^2 - x + 5$ by $x + 2$. Here $c = -2$.

-2	1	4	-1	5
		-2	-4	10
	1	2	-5	15

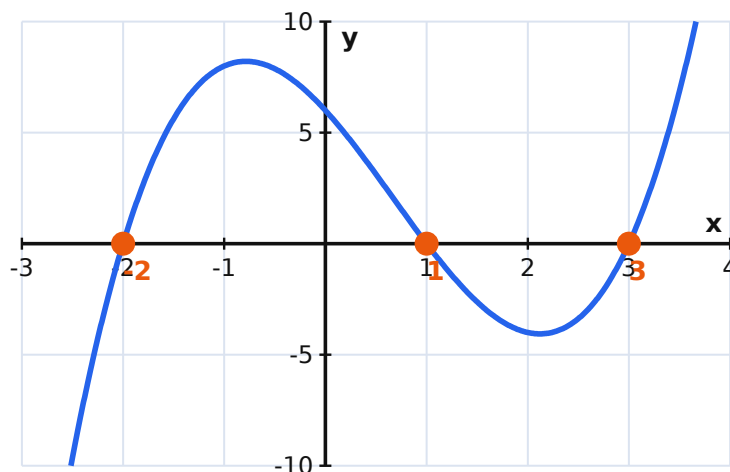
Bottom row: 1, 2, -5 and remainder 15, so $x^3+4x^2-x+5=(x+2)(x^2+2x-5)+15$.

What do the Remainder and Factor Theorems say?

Remainder Theorem and Factor Theorem

When $f(x)$ is divided by $x-c$, the remainder is $f(c)$. Therefore $x-c$ is a factor of $f(x)$ exactly when $f(c)=0$.

In the example, $f(-2)=-8+16+2+5=15$, the same as the remainder. You can find a remainder by substituting, without dividing.



The graph of $f(x) = x^3 - 2x^2 - 5x + 6$ crosses the axis at -2 , 1 and 3 , so $(x+2)$, $(x-1)$ and $(x-3)$ are factors.

How do you find all the zeros of a polynomial?

The Rational Root Theorem says that if a polynomial with integer coefficients has a rational zero $\frac{p}{q}$ in lowest terms, then p divides the constant term and q divides the leading coefficient.

p (factors of 6)	1, 2, 3, 6
q (factors of 2)	1, 2
p / q	1, 2, 3, 6, 1/2, 3/2

Candidates for $2x^3-3x^2-11x+6$: factors of 6 over factors of 2.

1. List the candidates $\frac{p}{q}$.
2. Test candidates with synthetic division until the remainder is 0.
3. Factor the quotient, using the quadratic formula if needed.
4. Collect all the zeros.

For $2x^3 - 3x^2 - 11x + 6$, the value $x = 3$ gives $54 - 27 - 33 + 6 = 0$. Division leaves $2x^2 + 3x - 2 = (2x - 1)(x + 2)$, so the zeros are $3, \frac{1}{2}$ and -2 .

3	2	-3	-11	6
		6	9	-6
	2	3	-2	0

Synthetic division by $x - 3$ leaves quotient $2x^2 + 3x - 2$ and remainder 0.

Questions about polynomial division and the factor theorem

When can you use synthetic division?

When the divisor is linear with leading coefficient 1, such as $x - 3$ or $x + 2$. For $x + 2$, use $c = -2$. For other divisors, use long division.

What does a zero remainder mean?

It means the divisor divides the polynomial evenly, so the divisor is a factor and c is a zero of the polynomial.

Why must I include zeros for missing terms?

Each position in the coefficient list stands for one power of x . Skipping a missing power would shift every later coefficient to the wrong place.

How do I find the remainder without dividing?

Substitute c into the polynomial. The value $f(c)$ is the remainder when dividing by $x - c$.

Does the Rational Root Theorem find irrational zeros?

No. It only lists possible rational zeros. After finding a rational zero and dividing, you may need the quadratic formula to find the remaining irrational or complex zeros.