

Precalculus - Polynomial Division and the Factor Theorem - Answer Key

Common Core A-APR.B.2, A-APR.D.6, A-APR.B.3 · maths-pdf.fr/en/

Use this key to check each division and zero. The synthetic division tables show every step, and final answers are in bold.

Precalculus · Common Core State Standards: **A-APR.B.2, A-APR.D.6, A-APR.B.3**

Exercise 1 - Divide a quadratic

$x^2 \div x = x$; subtract $x^2 + 3x$ to get $4x + 12$; $4x \div x = 4$; subtract $4x + 12$ to get 0.

$x + 4$ with remainder 0

Exercise 2 - Long division, leading coefficient 2

$2x^2 \div x = 2x$; subtract $2x^2 + 6x$ to get $-x - 3$; $-x \div x = -1$; subtract $-x - 3$ to get 0.

$2x - 1$

Exercise 3 - Remainder from long division

1	1	2	-4	7
		1	3	-1
	1	3	-1	6

Coefficients 1, 2, -4, 7 with $c = 1$.

The bottom row is 1, 3, -1 with remainder 6.

$$x^2 + 3x - 1 + \frac{6}{x-1}$$

Exercise 4 - Synthetic division

2	1	-4	1	6
		2	-4	-6
	1	-2	-3	0

$$c = 2.$$

The bottom row is 1, -2, -3, 0.

$$x^2 - 2x - 3, \text{ remainder } 0$$

Exercise 5 - Synthetic division with $c = -1$

-1	2	-3	5	-1
		-2	5	-10
	2	-5	10	-11

$$c = -1.$$

The bottom row is 2, -5, 10, -11.

$$2x^2 - 5x + 10 - \frac{11}{x+1}$$

Exercise 6 - Remainder Theorem

The remainder is $f(2) = 16 - 12 + 5$.

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Exercise 7 - Is it a factor?

$f(3) = 27 - 27 = 0$, so yes. Division gives $x^2 + 3x + 9$, which does not factor over the integers (discriminant $9 - 36 < 0$).

$$x^3 - 27 = (x - 3)(x^2 + 3x + 9)$$

Exercise 8 - Test a factor

$$f(-2) = -8 + 12 + 8 - 12 = 0.$$

Yes, because the remainder is 0

Exercise 9 - Find an unknown coefficient

$$f(2) = 8 + 4k - 8 - 4 = 4k - 4 = 0, \text{ so } k = 1.$$

$$k = 1$$

Exercise 10 - Factor completely

Synthetic division with $c=1$ on $1, 0, -7, 6$ gives $1, 1, -6$ with remainder 0, so
 $x^3 - 7x + 6 = (x-1)(x^2 + x - 6) = (x-1)(x+3)(x-2).$

$$(x-1)(x+3)(x-2)$$

Exercise 11 - Rational root candidates

$p \in \{\pm 1, \pm 2, \pm 3, \pm 6\}$ and $q \in \{\pm 1, \pm 2\}$. Dividing gives twelve candidates.

$$\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$$

Exercise 12 - Find all zeros

3	2	-3	-11	6
		6	9	-6
	2	3	-2	0

$c = 3$ leaves 2, 3, -2 and remainder 0.

The quotient is $2x^2 + 3x - 2 = (2x-1)(x+2).$

$$x = 3, \frac{1}{2}, -2$$

Exercise 13 - Missing terms

2	1	0	0	-8
		2	4	8
	1	2	4	0

Placeholders 0 for x^2 and x .

The bottom row is 1, 2, 4, 0.

x^2+2x+4 , remainder 0

Exercise 14 - Biquadratic

Let $u = x^2$: $u^2 - 5u + 4 = (u-1)(u-4) = 0$, so $x^2 = 1$ or $x^2 = 4$.

$x = \pm 1, \pm 2$

Exercise 15 - Quartic zeros

Divide by $x-1$: $x^3 - 7x - 6$. Divide by $x+1$: $x^2 - x - 6 = (x-3)(x+2)$.

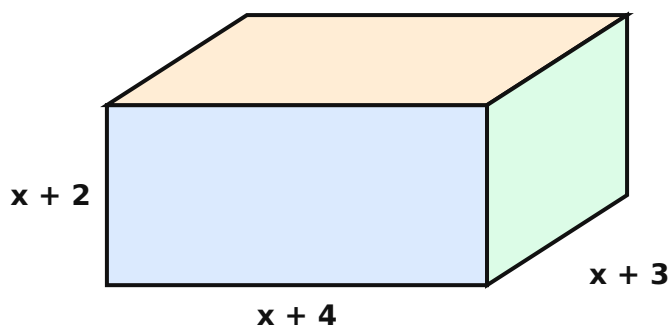
$x = 1, -1, 3, -2$

Exercise 16 - Divide by a non-monic linear divisor

$6x^2 \div 2x = 3x$; subtract $6x^2 + 9x$ to get $-8x - 5$; $-8x \div 2x = -4$; subtract $-8x - 12$ to get 7.

$3x - 4 + \frac{7}{2x+3}$

Exercise 17 - Box dimensions



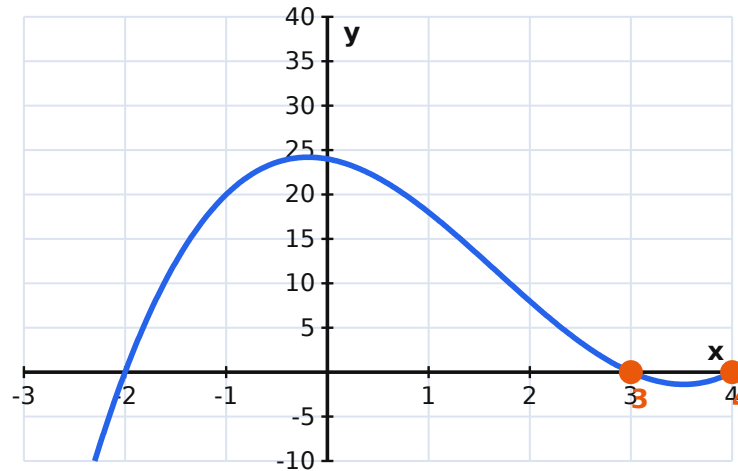
Dimensions: $x + 2$, $x + 3$, $x + 4$.

Synthetic division with $c = -2$: 1, 9, 26, 24 $\rightarrow$ 1, 7, 12, remainder 0. The quotient

$x^2 + 7x + 12 = (x+3)(x+4)$.

$x+3$ and $x+4$ feet

Exercise 18 - Break-even years



Zeros at $t = 3$ and $t = 4$.

$P(4) = 64 - 80 - 8 + 24 = 0$. Division by $t - 4$ gives $t^2 - t - 6 = (t - 3)(t + 2)$. The zeros are 4, 3, -2; only $t \geq 0$ counts.

$t = 3$ and $t = 4$ years

Exercise 19 - Multiplicity of a zero

$f\left(\frac{1}{2}\right) = \frac{1}{2} - 2 + \frac{5}{2} - 1 = 0$. Factoring gives $(2x - 1)^2(x - 1)$.

$\frac{1}{2}$ (multiplicity 2) and 1 (multiplicity 1)

Exercise 20 - Polynomial with integer coefficients

$f(x) = (x - 2)(2x + 1)(x - 3)$. First $(x - 2)(x - 3) = x^2 - 5x + 6$, then multiply by $(2x + 1)$: $2x^3 - 9x^2 + 7x + 6$.

$2x^3 - 9x^2 + 7x + 6$