

Precalculus - Polynomial Functions and Their Graphs - Lesson

Common Core F-IF.C.7c, A-APR.B.3, F-IF.B.4 · maths-pdf.fr/en/

A polynomial function has whole-number exponents, a degree n and a leading coefficient. Its end behavior depends on whether n is even or odd and on the sign of the leading coefficient. Each zero r gives a factor $(x - r)$; the graph crosses the x -axis at odd multiplicity and touches it at even multiplicity.

Precalculus · Common Core State Standards: **F-IF.C.7c, A-APR.B.3, F-IF.B.4**

Key facts

- Degree n : at most n real zeros and at most $n - 1$ turning points.
- Even degree: both ends point the same way; odd degree: opposite ways.
- Positive leading coefficient: right end rises. Negative: right end falls.
- Odd multiplicity crosses the axis; even multiplicity touches it.
- $f(0)$ gives the y -intercept; a sign chart solves polynomial inequalities.

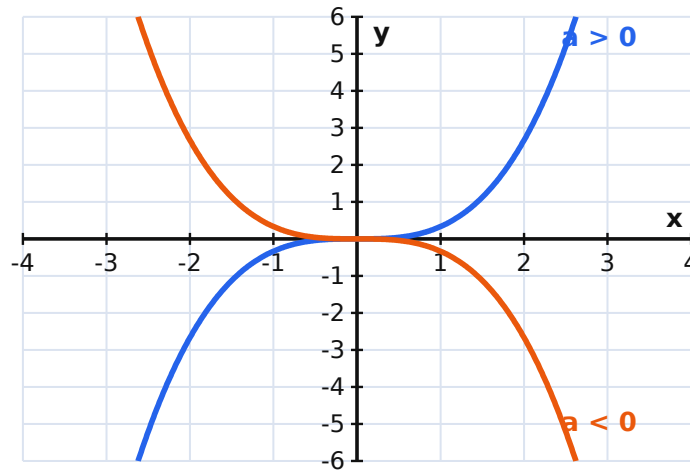
What is a polynomial function?

A polynomial function has the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + (\text{lower terms}) + a_0$ with whole-number exponents. The largest exponent n is the **degree**, and a_n is the **leading coefficient**. The graph is smooth and continuous with no breaks, corners or gaps. A polynomial of degree n has at most $n - 1$ turning points and at most n real zeros.

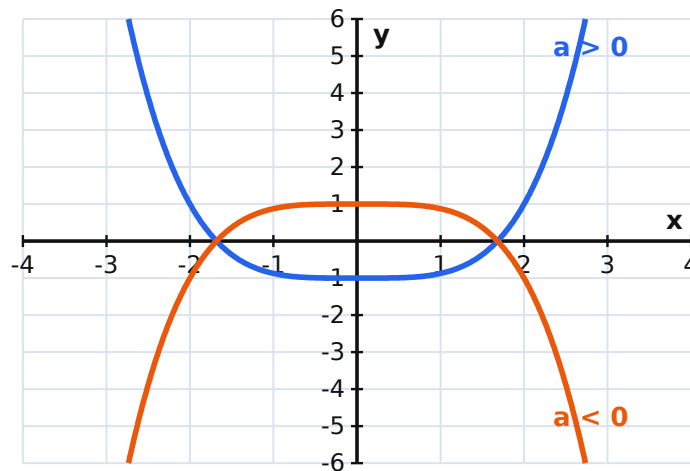
How do you find end behavior?

For large $|x|$, the term $a_n x^n$ dominates. Check the degree and the sign of the leading coefficient.

Degree	Leading coefficient	Left end	Right end
Even	Positive	Up	Up
Even	Negative	Down	Down
Odd	Positive	Down	Up
Odd	Negative	Up	Down



Odd degree: the ends point in opposite directions.



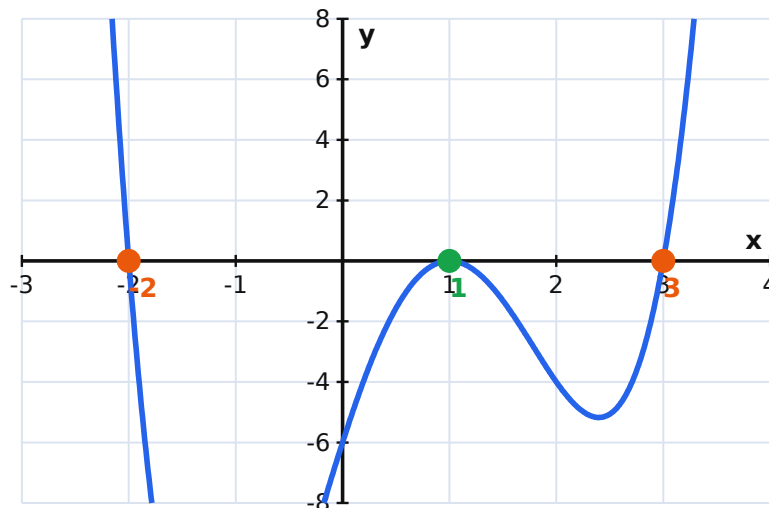
Even degree: the ends point in the same direction.

How do zeros and multiplicity shape the graph?

A **zero** of f is an input where $f(x)=0$. Each zero r corresponds to a factor $(x-r)$. The number of times the factor appears is the **multiplicity**. If the multiplicity is odd, the graph crosses the x -axis at r . If it is even, the graph touches the axis and turns around.

Factor theorem preview

If r is a zero of f , then $(x-r)$ is a factor of $f(x)$, and the reverse holds.



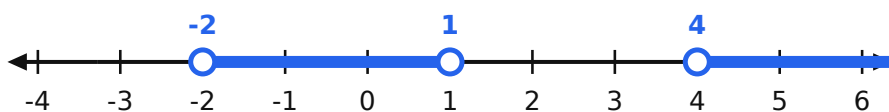
$f(x) = (x+2)(x-1)^2(x-3)$: the graph crosses at -2 and 3 , and touches at 1 .

How do you sketch a polynomial from factored form?

1. Find the zeros and their multiplicities.
2. Plot the y -intercept $f(0)$.
3. Use the degree and leading coefficient for end behavior.
4. Connect the pieces: cross at odd multiplicities, touch at even ones.

Example: For $f(x) = (x+2)(x-1)^2(x-3)$, the degree is 4 with leading coefficient 1, so both ends point up. The y -intercept is $f(0) = (2)(1)(-3) = -6$.

To solve $f(x) > 0$ or $f(x) < 0$, list the zeros on a number line and test one value in each interval. The sign can change only at a zero of odd multiplicity.



Sign chart for $(x+2)(x-1)(x-4) > 0$: positive on $(-2, 1)$ and $(4, \infty)$.

Polynomial models appear in many places. The volume of a box folded from a sheet of cardboard is a cubic in the side of the cut corner, and the path of a roller coaster hill can be modeled by a cubic with three zeros. In both cases the zeros mark where the model starts and ends, which tells you the realistic domain. Always check that the answer you find lies in that domain before you trust it, and remember that a model is only valid on the interval where its inputs make sense.

Questions about polynomial functions and their graphs

What is the degree of a polynomial?

The degree is the highest exponent of the variable after the polynomial is simplified. The degree of $5x^3 - 2x^2 + 1$ is 3.

How many turning points can a polynomial have?

At most one fewer than its degree. A cubic has at most 2 turning points, and a quartic has at most 3.

What does multiplicity tell you?

It tells how many times a factor repeats. Odd multiplicity means the graph crosses at that zero, and even multiplicity means it touches and turns around.

How do you find the y-intercept?

Evaluate $f(0)$. In factored form, substitute 0 for x in every factor and multiply.

Why does end behavior depend only on the leading term?

For very large $|x|$, the highest power grows much faster than all the lower powers, so $a_n x^n$ controls the sign and size of $f(x)$.