

Precalculus - Polynomial Functions and Their Graphs - Answer Key

Common Core F-IF.C.7c, A-APR.B.3, F-IF.B.4 · maths-pdf.fr/en/

Use this key to check each polynomial problem. Zeros, multiplicities and sign tests are shown in steps, with final answers in bold.

Precalculus · Common Core State Standards: **F-IF.C.7c, A-APR.B.3, F-IF.B.4**

Exercise 1 - Degree and leading coefficient

The highest power is x^3 with coefficient 5.

Degree 3; leading coefficient 5

Exercise 2 - Expand a product

$(x-2)(x+3) = x^2 + x - 6$. Multiply by $(x+1)$: $x^3 + 2x^2 - 5x - 6$.

$$x^3 + 2x^2 - 5x - 6$$

Exercise 3 - End behavior, odd degree

Odd degree with a negative leading coefficient: left end up, right end down.

As $x \rightarrow -\infty$, $f(x) \rightarrow \infty$; as $x \rightarrow \infty$, $f(x) \rightarrow -\infty$

Exercise 4 - End behavior, even degree

Even degree with a positive leading coefficient: both ends up.

$$f(x) \rightarrow \infty \text{ as } x \rightarrow \pm\infty$$

Exercise 5 - Zeros from factors

Set each factor equal to 0.

$$x=0, 4, -5$$

Exercise 6 - Multiplicity

$x=1$ has multiplicity 2 (even): touches and turns. $x=-3$ has multiplicity 3 (odd): crosses with a flattening. Degree $=2+3=5$.

1 (mult. 2, touch); -3 (mult. 3, cross); degree 5

Exercise 7 - Factor out x

$$x^3-9x = x(x^2-9) = x(x-3)(x+3).$$

$$x=0, 3, -3$$

Exercise 8 - Factor by grouping

$$x^2(x+2)-9(x+2) = (x+2)(x^2-9) = (x+2)(x-3)(x+3).$$

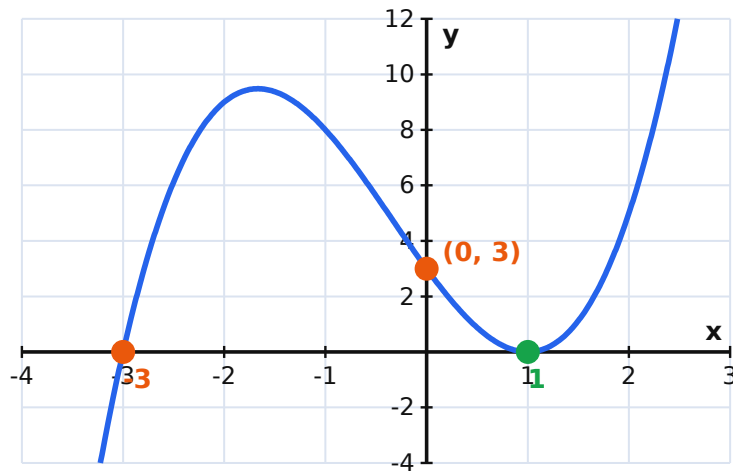
$$x=-2, 3, -3$$

Exercise 9 - Build a polynomial

$$f(x) = a(x+1)(x-2)(x-4). \quad f(0) = a(1)(-2)(-4) = 8a = 16, \text{ so } a = 2. \text{ Expand: } 2x^3 - 10x^2 + 4x + 16.$$

$$f(x) = 2(x+1)(x-2)(x-4) = 2x^3 - 10x^2 + 4x + 16$$

Exercise 10 - Read a graph



Factors $(x + 3)$ and $(x - 1)^2$.

Crossing at -3 : factor $(x+3)$. Touching at 1 : factor $(x-1)^2$. The right end rises, so $a > 0$. With y -intercept 3 : $a(3)(1) = 3$, $a = 1$.

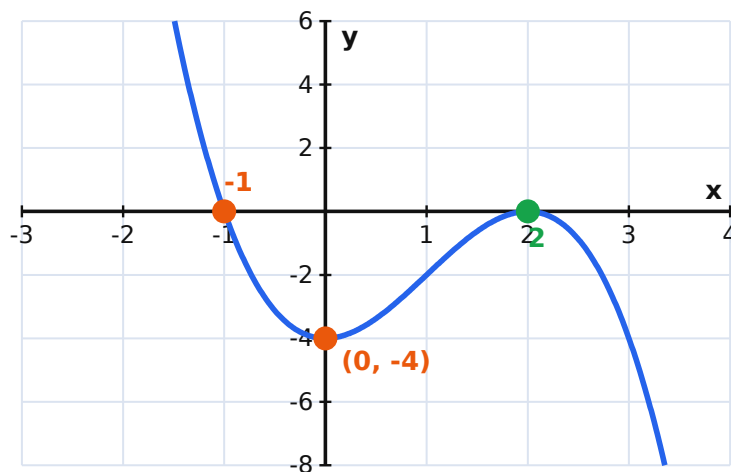
$$f(x) = (x+3)(x-1)^2$$

Exercise 11 - Turning points

Turning points are at most $n-1$, so a degree-6 polynomial has at most 5. With 3 turning points, $n-1 \geq 3$, so $n \geq 4$.

(a) 5; (b) 4

Exercise 12 - Sketch a polynomial

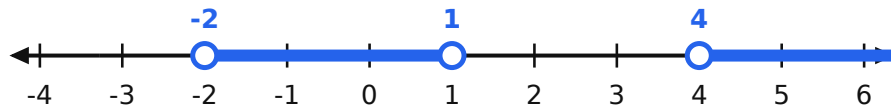


Sketch of f .

$f(0) = -(1)(4) = -4$. Degree 3 with a negative leading coefficient: left up, right down. At -1 the graph crosses; at 2 it touches.

y-int -4 ; left up, right down; cross at -1 , touch at 2

Exercise 13 - Solve a polynomial inequality



The product is positive on $(-2, 1)$ and $(4, \infty)$.

Test values: $x = -3$: $(-1)(-4)(-7) < 0$. $x = 0$: $(2)(-1)(-4) = 8 > 0$. $x = 2$: $(4)(1)(-2) < 0$. $x = 5$: $(7)(4)(1) > 0$.

$(-2, 1) \cup (4, \infty)$

Exercise 14 - Intermediate Value Theorem

$f(1) = 1 - 1 - 1 = -1 < 0$ and $f(2) = 8 - 2 - 1 = 5 > 0$. A polynomial is continuous, so it must cross 0 between 1 and 2.

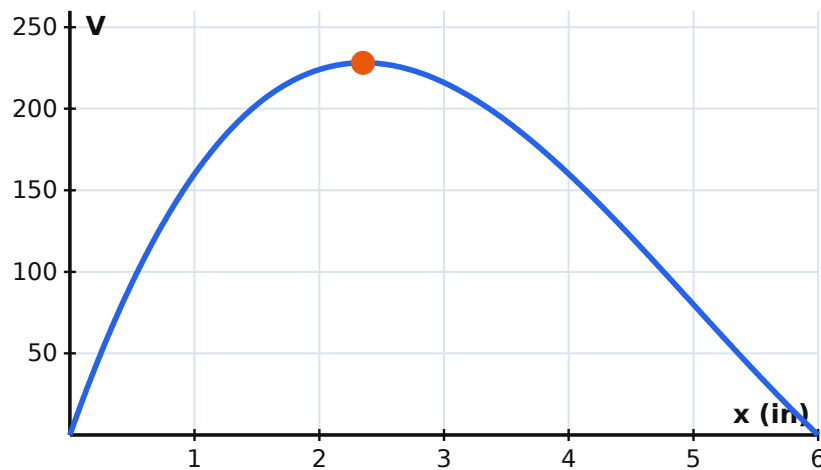
A zero exists in $(1, 2)$

Exercise 15 - Evaluate a polynomial

$$2(81) - 3(27) + 3 - 5 = 162 - 81 + 3 - 5.$$

79

Exercise 16 - Box volume

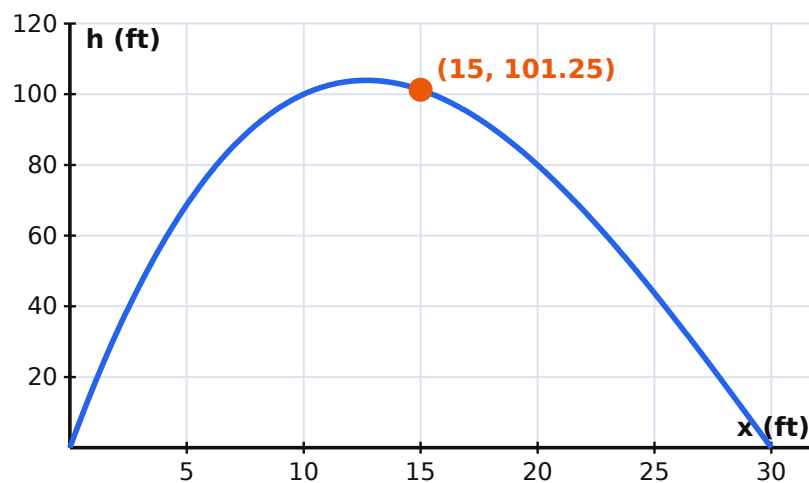


The peak is between $x = 2$ and $x = 3$.

$V(1) = 1(10)(16) = 160$. $V(2) = 2(8)(14) = 224$. $V(3) = 3(6)(12) = 216$. The sides $12 - 2x$ and $18 - 2x$ must be positive, so $0 < x < 6$.

160, 224, 216 **in³**; $0 < x < 6$; $x = 2$

Exercise 17 - Roller coaster



Height peaks near the middle.

$h(15) = 0.01(15)(-15)(-45) = 101.25$. $h(10) = 0.01(10)(-20)(-50) = 100$. The zeros are where the track is at ground level, the start and end of the hill.

101.25 **ft**; 100 **ft**; **ground level at both ends**

Exercise 18 - Zeros with multiplicity

$f(x) = a(x-1)^2(x+2)(x-3)$. $f(0) = a(1)(2)(-3) = -6a = 6$, so $a = -1$. Expanded:
 $-x^4 + 3x^3 + 3x^2 - 11x + 6$.

$$f(x) = -(x-1)^2(x+2)(x-3) = -x^4 + 3x^3 + 3x^2 - 11x + 6$$

Exercise 19 - Least-degree equation

$f(x) = a(x+3)^2(x-2)$. $f(0) = a(9)(-2) = -18a = -18$, so $a = 1$. Degree 3 with $a > 0$ rises on the right.

$$f(x) = (x+3)^2(x-2)$$

Exercise 20 - Find all zeros

$(x-1)$ is a factor: $x^3 - 6x^2 + 11x - 6 = (x-1)(x^2 - 5x + 6) = (x-1)(x-2)(x-3)$.

$$x = 1, 2, 3$$