

Precalculus - Rational Functions and Asymptotes

- Answer Key

Common Core A-APR.D.7, F-IF.C.7d, A-CED.A.1 · maths-pdf.fr/en/

Use this key to check each rational function. Factoring, cancelling and the extraneous-solution checks are shown, with final answers in bold.

Precalculus · Common Core State Standards: **A-APR.D.7, F-IF.C.7d, A-CED.A.1**

Exercise 1 - Domain

$$x^2 - 5x + 6 = (x-2)(x-3), \text{ so } x \neq 2 \text{ and } x \neq 3.$$

$$(-\infty, 2) \cup (2, 3) \cup (3, \infty)$$

Exercise 2 - Hole and asymptote

$f(x) = \frac{(x-3)(x+3)}{(x-3)(x+2)}$. The factor $x-3$ cancels: hole at $x=3$ with $y = \frac{3+3}{3+2} = \frac{6}{5}$. The factor $x+2$ remains: asymptote $x = -2$.

Hole $(3, \frac{6}{5})$; **vertical asymptote** $x = -2$

Exercise 3 - Equal degrees

Equal degrees: divide leading coefficients, $\frac{6}{2}$.

$$y = 3$$

Exercise 4 - Lower numerator degree

The numerator has degree 1 and the denominator has degree 2.

$$y = 0$$

Exercise 5 - No horizontal asymptote

The degree difference is $4 - 2 = 2$, more than 1.

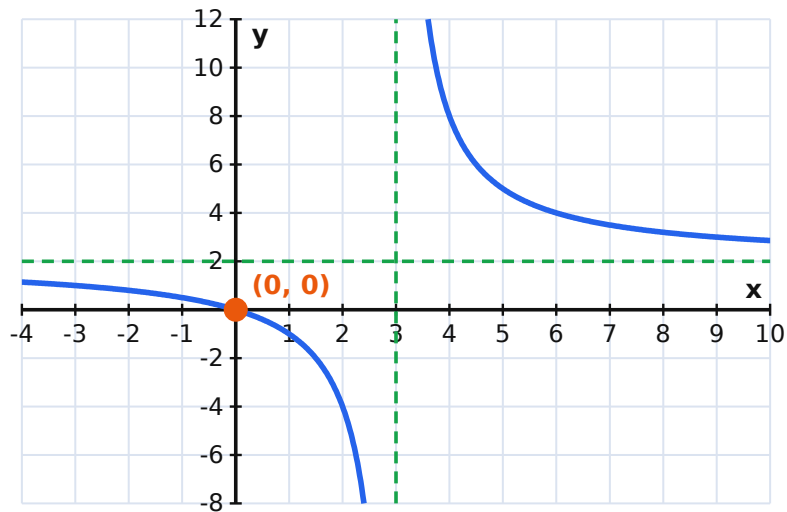
Neither

Exercise 6 - Intercepts

$2x-6=0$ gives $x=3$. $f(0)=\frac{-6}{4}=-\frac{3}{2}$.

x-intercept $(3,0)$; **y-intercept** $(0,-\frac{3}{2})$

Exercise 7 - Read a graph

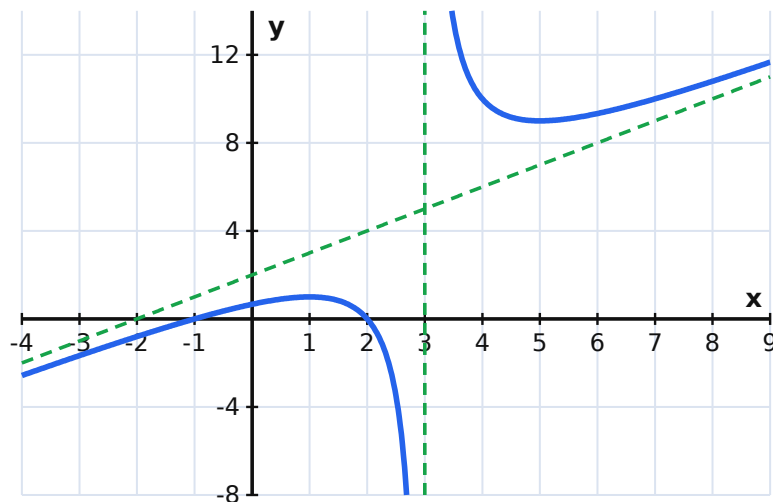


Asymptotes $x = 3$ and $y = 2$.

The denominator is 0 at $x=3$. The degrees are equal with leading coefficients 2 and 1.
 $f(0)=0$.

$x=3$; $y=2$; **intercept** $(0,0)$

Exercise 8 - Slant asymptote

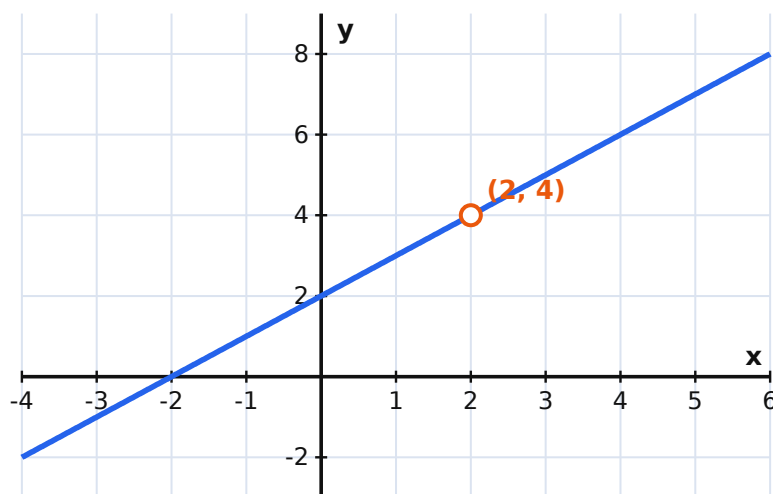


The dashed slanted line is $y = x + 2$.

Divide with synthetic division, $c = 3$: $1, -1, -2 \rightarrow 1, 2, 4$. The quotient is $x + 2$.

$$y = x + 2$$

Exercise 9 - Find the hole



Hole at $(2, 4)$.

Cancel $x - 2$: $f(x) = x + 2$. At $x = 2$, $y = 4$.

$(2, 4)$

Exercise 10 - Solve a proportion

Cross multiply: $3(x+2)=2(x-1)$, $3x+6=2x-2$, so $x=-8$. Check: $\frac{3}{-9}=-\frac{1}{3}$ and $\frac{2}{-6}=-\frac{1}{3}$.

$$x = -8$$

Exercise 11 - Solve with two fractions

Multiply by $x(x+1)$: $2(x+1)+3x=2x(x+1)$, so $5x+2=2x^2+2x$ and $2x^2-3x-2=0$. Factor: $(2x+1)(x-2)=0$. Both $x=-\frac{1}{2}$ and $x=2$ keep the denominators nonzero.

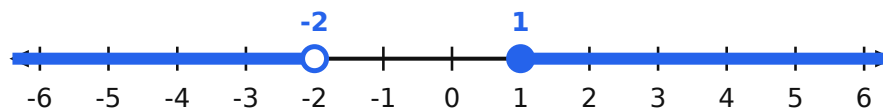
$$x = 2 \text{ or } x = -\frac{1}{2}$$

Exercise 12 - Extraneous solution

Multiply by $x-3$: $x=3+2(x-3)=2x-3$, so $x=3$. But $x=3$ makes the denominators 0.

No solution (3 is extraneous)

Exercise 13 - Rational inequality



Solution: left of -2 and from 1 rightward.

Critical values: $x=1$ (numerator 0, included) and $x=-2$ (denominator 0, excluded). Test $x=-3$: $\frac{-4}{-1}>0$. $x=0$: $\frac{-1}{2}<0$. $x=2$: $\frac{1}{4}>0$.

$$(-\infty, -2) \cup [1, \infty)$$

Exercise 14 - Write an equation

The denominator is $x-2$. Equal degrees with ratio 3: numerator $3(x+1)$.

$$f(x) = \frac{3(x+1)}{x-2}$$

Exercise 15 - Behavior near an asymptote

$$f(2.01) = \frac{1}{0.01} = 100. \quad f(1.99) = \frac{1}{-0.01} = -100.$$

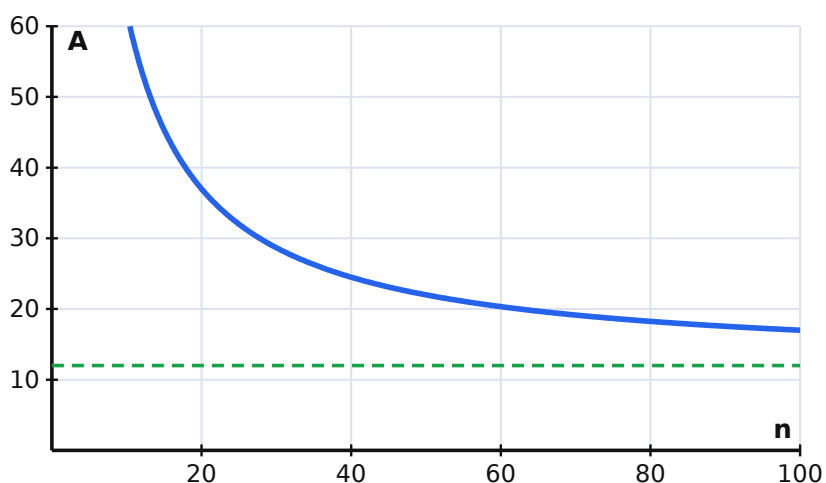
100 and -100 ; $f \rightarrow \infty$ as $x \rightarrow 2^+$, $f \rightarrow -\infty$ as $x \rightarrow 2^-$

Exercise 16 - Hole and asymptote together

Cancel $x-1$: $\frac{x+3}{x-5}$. The hole is at $x=1$, $y=\frac{4}{-4}=-1$. The asymptote is $x=5$.

Hole $(1,-1)$; **asymptote** $x=5$

Exercise 17 - Average cost of shirts

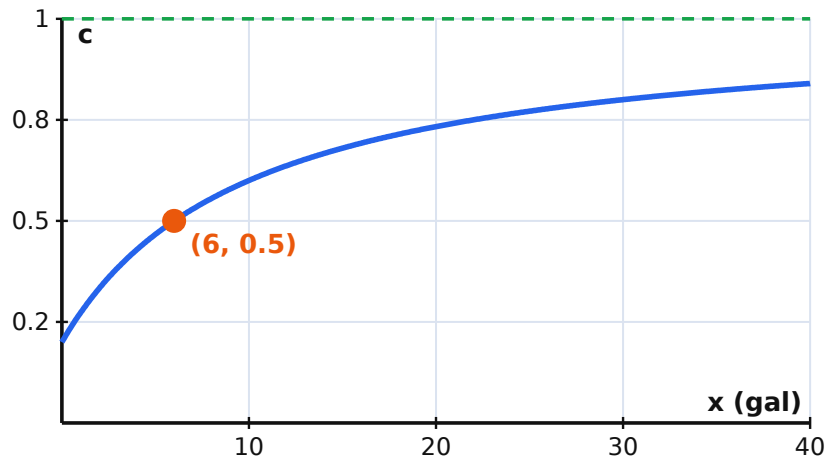


The average cost falls toward 12 dollars.

$A(50) = \frac{1100}{50} = 22$. The asymptote $y=12$: as n grows, the fixed cost is spread out and the average approaches \$12. Solve $500+12n \leq 15n$: $n \geq \frac{500}{3} \approx 166.7$.

\$22; $y=12$; $n=167$ shirts

Exercise 18 - Salt mixture



Half salt at $x = 6$.

Equal degrees, ratio $\frac{1}{1}$: $y=1$ (the fraction can never reach 100%). Solve $2+x=0.5(10+x)$:
 $2+x=5+0.5x$, $0.5x=3$, $x=6$.

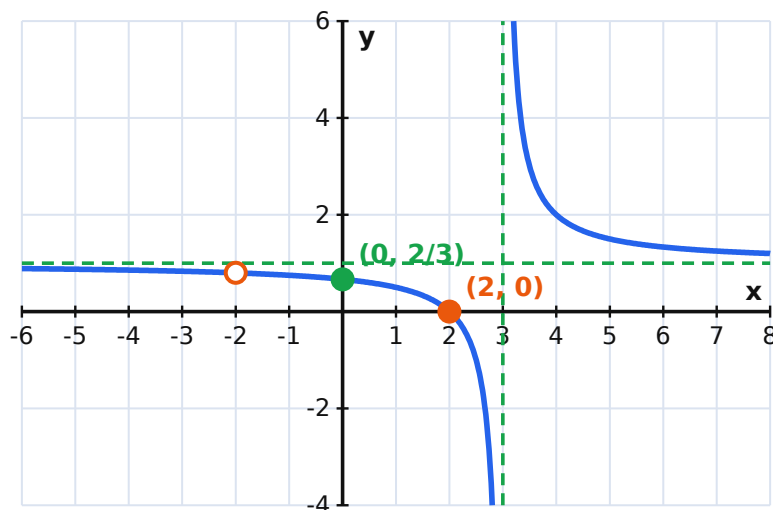
$y=1$; $x=6$ gallons

Exercise 19 - Painting a fence

Multiply by $2x(x-3)$: $2(x-3)+2x=x(x-3)$, so $4x-6=x^2-3x$ and $x^2-7x+6=0$, giving $x=6$ or $x=1$. If $x=1$, then $x-3<0$, which is impossible. So $x=6$. Check: $\frac{1}{6}+\frac{1}{3}=\frac{1}{2}$.

Mei 6 hours; Jamal 3 hours

Exercise 20 - Complete analysis



Hole at $(-2, 0.8)$, asymptotes $x = 3$ and $y = 1$.

$f(x) = \frac{(x-2)(x+2)}{(x-3)(x+2)}$. Cancel $x+2$: hole at $x = -2$, $y = \frac{-4}{-5} = \frac{4}{5}$. Vertical asymptote $x = 3$. Equal degrees: $y = 1$. x -intercept: 2. y -intercept: $f(0) = \frac{-4}{-6} = \frac{2}{3}$.

Hole $\left(-2, \frac{4}{5}\right)$; $x = 3$; $y = 1$; $(2, 0)$; $\left(0, \frac{2}{3}\right)$