

Precalculus - Sequences and Series - Lesson

Common Core F-IF.A.3, F-BF.A.2, A-SSE.B.4 · maths-pdf.fr/en/

A sequence is an ordered list of numbers. An arithmetic sequence adds a common difference d , so $a_n = a_1 + (n - 1)d$. A geometric sequence multiplies by a common ratio r , so $a_n = a_1 r^{(n-1)}$. An infinite geometric series with $|r| < 1$ has the sum $a_1/(1 - r)$.

Precalculus · Common Core State Standards: **F-IF.A.3, F-BF.A.2, A-SSE.B.4**

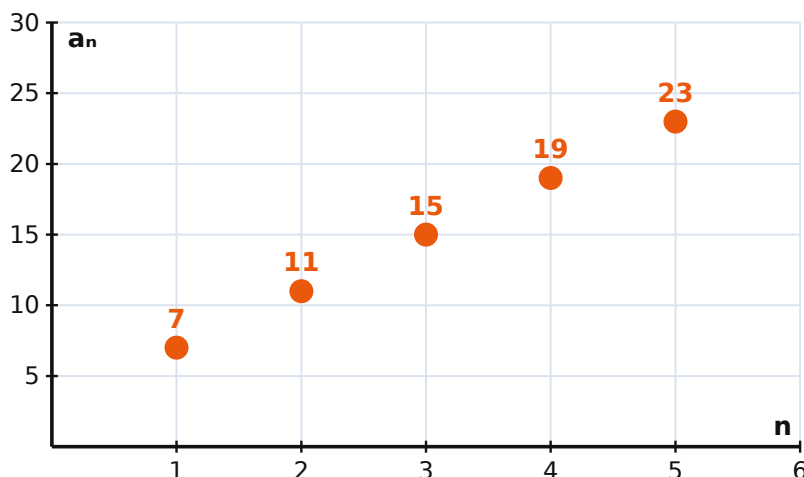
Key facts

- Arithmetic: $a_n = a_1 + (n - 1)d$; sum $S_n = (n/2)(a_1 + a_n)$.
- Geometric: $a_n = a_1 r^{(n-1)}$; sum $S_n = a_1(1 - r^n)/(1 - r)$.
- Sigma notation adds the terms from the lower index to the upper index.
- An infinite geometric series converges only if $|r| < 1$, and then $S = a_1/(1 - r)$.
- A recursive formula gives each term from the one before it.

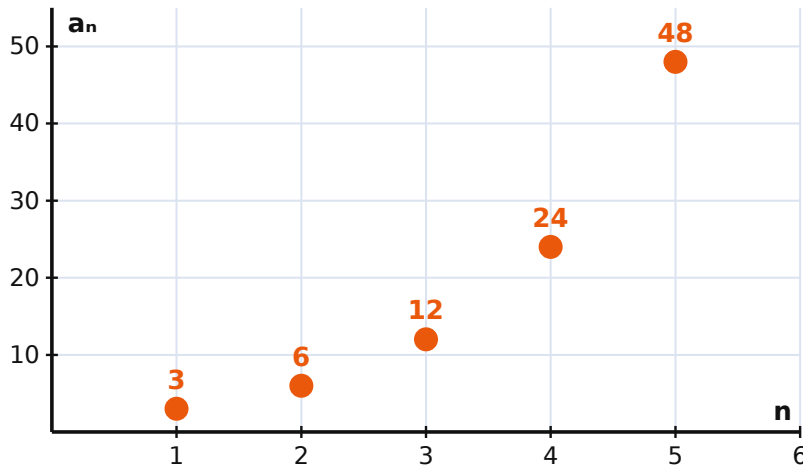
What is a sequence?

A sequence is an ordered list of numbers $a_1, a_2, a_3, \dots$. An **explicit formula** gives a_n directly from n , and a **recursive formula** gives each term from the previous one. A sequence is a function whose domain is the positive integers, so its graph is a set of separate points.

Type	Pattern	Explicit formula	Recursive formula
Arithmetic	add the common difference d	$a_n = a_1 + (n-1)d$	$a_n = a_{n-1} + d$
Geometric	multiply by the common ratio r	$a_n = a_1 r^{n-1}$	$a_n = r a_{n-1}$



The points of 7,11,15,19,23 lie on a line: $a_n = 4n + 3$.



The points of 3,6,12,24,48 grow by a factor of 2: $a_n = 3 \cdot 2^{n-1}$.

To decide which type you have, test the pattern. Subtract each term from the next: if every difference is the same, the sequence is arithmetic. Divide each term by the one before it: if every ratio is the same, it is geometric. For 5,10,20,40 the differences are 5,10,20, which are not equal, but every ratio is 2. Many real situations follow these patterns: a salary that rises by a fixed raise each year is arithmetic, while a savings account earning a fixed percent or a ball that rebounds to a fixed fraction of its height is geometric. A recursive formula such as $a_n = a_{n-1} + 4$ with $a_1 = 5$ describes the same arithmetic sequence as the explicit formula $a_n = 4n + 1$.

What is a series and how do you sum it?

A series is the sum of the terms of a sequence. Sigma notation $\sum_{k=1}^n a_k$ means $a_1 + a_2 + \dots + a_n$, where k runs from the bottom number to the top number. For example $\sum_{k=1}^5 (2k+1) = 3+5+7+9+11 = 35$.

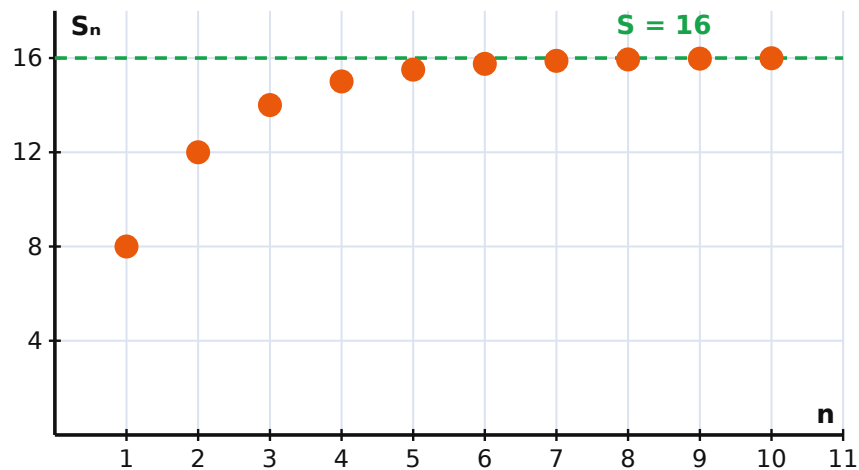
Finite sum formulas

Arithmetic: $S_n = \frac{n}{2}(a_1 + a_n)$. Geometric ($r \neq 1$): $S_n = a_1 \frac{1-r^n}{1-r}$.

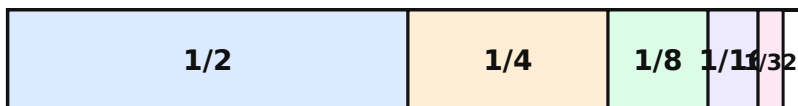
The arithmetic formula pairs the first and last terms: each pair sums to $a_1 + a_n$, and there are $\frac{n}{2}$ pairs. For $7+11+15+\dots$ with 20 terms, $a_{20} = 83$ and $S_{20} = 10(7+83) = 900$.

When does an infinite series have a sum?

An infinite geometric series converges only when $|r| < 1$, and then $S = \frac{a_1}{1-r}$. For $8+4+2+1+\dots$, $a_1 = 8$ and $r = \frac{1}{2}$, so $S = \frac{8}{1/2} = 16$. If $|r| \geq 1$, the partial sums do not settle down and the series diverges.



The partial sums approach 16 but never pass it.



$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ gets closer and closer to 1

Geometric picture: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ fills a bar of length 1.

Infinite series explain repeating decimals. Since $0.\overline{36} = 0.36 + 0.0036 + 0.000036 + \dots$ is geometric with $a_1 = 0.36$ and $r = 0.01$, its sum is $\frac{0.36}{0.99} = \frac{4}{11}$.

Questions about sequences and series

How do you tell if a sequence is arithmetic or geometric?

Subtract consecutive terms: if the differences are constant, it is arithmetic. Divide consecutive terms: if the ratios are constant, it is geometric.

What does sigma notation mean?

The symbol sigma means sum. The sum from $k = 1$ to 5 of $(2k + 1)$ adds the values of $2k + 1$ for $k = 1, 2, 3, 4, 5$.

When does an infinite geometric series have a sum?

When the absolute value of the ratio is less than 1. Then the terms shrink fast enough, and the sum is the first term divided by 1 minus r .

What is the difference between a sequence and a series?

A sequence is a list of numbers, such as 2, 4, 8, 16. A series is the sum of those terms, such as

$$2 + 4 + 8 + 16 = 30.$$

What is the difference between explicit and recursive formulas?

An explicit formula finds any term directly from n . A recursive formula needs the previous term, so you must start from the first term and work forward.