

Precalculus - Sequences and Series - Answer Key

Common Core F-IF.A.3, F-BF.A.2, A-SSE.B.4 · maths-pdf.fr/en/

Use this key to check each sequence and sum. Solutions identify the pattern, show the formula used and give the final answer in bold.

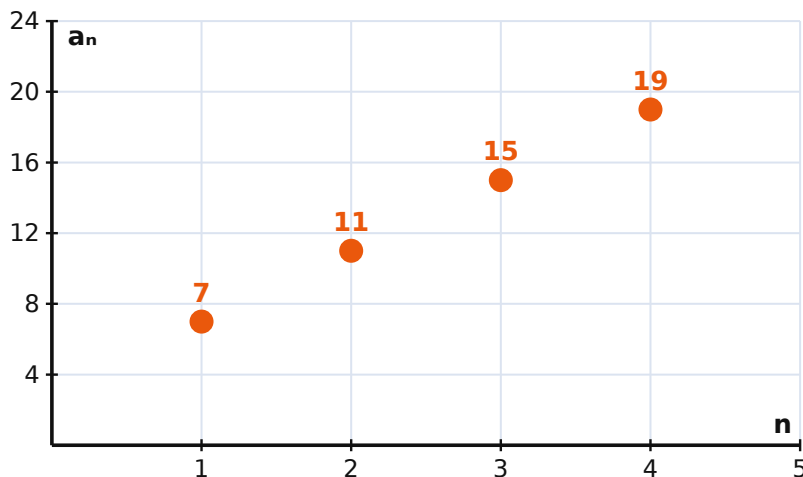
Precalculus · Common Core State Standards: **F-IF.A.3, F-BF.A.2, A-SSE.B.4**

Exercise 1 - List terms

$$a_1 = 5, a_2 = 8, a_3 = 11, a_4 = 14.$$

5, 8, 11, 14

Exercise 2 - Explicit arithmetic formula



Constant difference 4.

$$d = 4 \text{ and } a_1 = 7, \text{ so } a_n = 7 + 4(n-1) = 4n + 3. \quad a_{20} = 4(20) + 3 = 83.$$

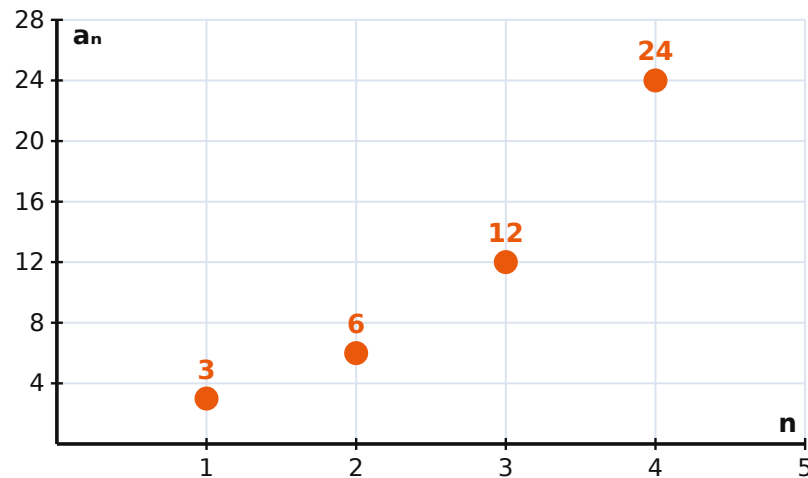
$$a_n = 4n + 3; \quad a_{20} = 83$$

Exercise 3 - Formula from two terms

$$5d = 25 - 10 = 15, \text{ so } d = 3. \quad a_1 = 10 - 2(3) = 4. \text{ Then } a_n = 4 + 3(n-1) = 3n + 1.$$

$$a_n = 3n + 1$$

Exercise 4 - Geometric formula



Common ratio 2.

$$r = 2, a_1 = 3. a_{10} = 3 \cdot 2^9 = 3 \cdot 512 = 1536.$$

$$a_n = 3 \cdot 2^{n-1}; a_{10} = 1536$$

Exercise 5 - Fractional ratio

$$r = \frac{27}{81} = \frac{1}{3}. a_6 = 81 \left(\frac{1}{3}\right)^5 = \frac{81}{243} = \frac{1}{3}.$$

$$r = \frac{1}{3}; a_6 = \frac{1}{3}$$

Exercise 6 - Recursive sequence

$$a_2 = 3(2) - 1 = 5. a_3 = 3(5) - 1 = 14. a_4 = 3(14) - 1 = 41.$$

5, 14, 41

Exercise 7 - Recursive to explicit

It is arithmetic with $d = 4$: $a_n = 5 + 4(n-1)$.

$$a_n = 4n + 1$$

Exercise 8 - Sum of the first 100 integers

$$S_{100} = \frac{100}{2}(1+100) = 50 \cdot 101.$$

5050

Exercise 9 - Arithmetic series

$$a_{30} = 5 + 29(3) = 92. \quad S_{30} = \frac{30}{2}(5+92) = 15 \cdot 97.$$

1455

Exercise 10 - Sum of 20 terms

$$a_{20} = 83. \quad S_{20} = \frac{20}{2}(7+83) = 10 \cdot 90.$$

900

Exercise 11 - Sigma notation

$$3+5+7+9+11.$$

35

Exercise 12 - Sigma with an arithmetic formula

$$\text{The first term is 1 and the last term is } 3(40)-2=118. \quad S_{40} = \frac{40}{2}(1+118) = 20 \cdot 119.$$

2380

Exercise 13 - Geometric series

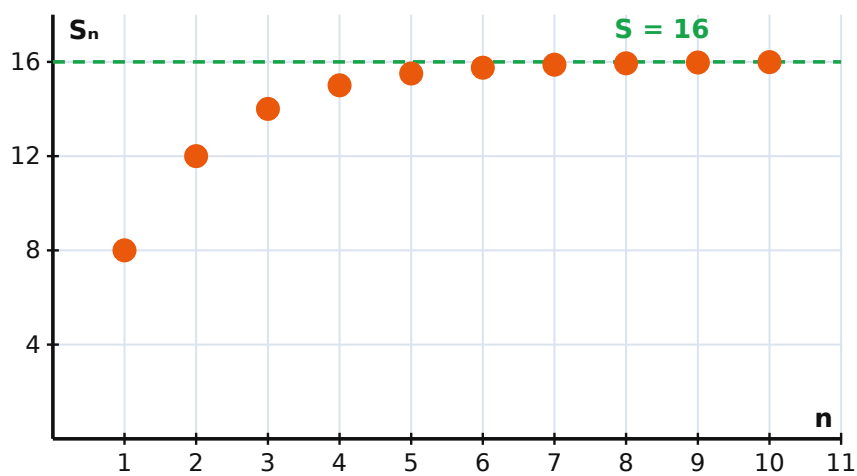
$$a_1 = 2, r = 3, n = 6. \quad S_6 = 2 \cdot \frac{1-3^6}{1-3} = 2 \cdot \frac{-728}{-2}.$$

728

Exercise 14 - Partial sum of a convergent series

$$a_1 = 1, r = \frac{1}{2}. \quad S_8 = \frac{1 - \left(\frac{1}{2}\right)^8}{1 - \frac{1}{2}} = \frac{255/256}{1/2} = \frac{255}{128} \approx 1.992.$$

Exercise 15 - Infinite geometric series



The partial sums approach 16.

$$r = \frac{1}{2}, \text{ so } |r| < 1. \quad S = \frac{8}{1 - \frac{1}{2}} = 16.$$

16

Exercise 16 - Sigma with infinity

$$a_1 = 3 \text{ and } r = 0.4. \quad S = \frac{3}{1 - 0.4} = 5.$$

5

Exercise 17 - Converge or diverge?

$$(a) \, r = 3, |r| \geq 1: \text{ diverges. } (b) \, r = -\frac{1}{2}: \text{ converges; } S = \frac{5}{1 + \frac{1}{2}} = \frac{10}{3}.$$

$$(a) \text{ no sum; } (b) \, \frac{10}{3}$$

Exercise 18 - Repeating decimal

$$0.36 + 0.0036 + \dots \text{ has } a_1 = 0.36 \text{ and } r = 0.01. \quad S = \frac{0.36}{0.99} = \frac{36}{99} = \frac{4}{11}.$$

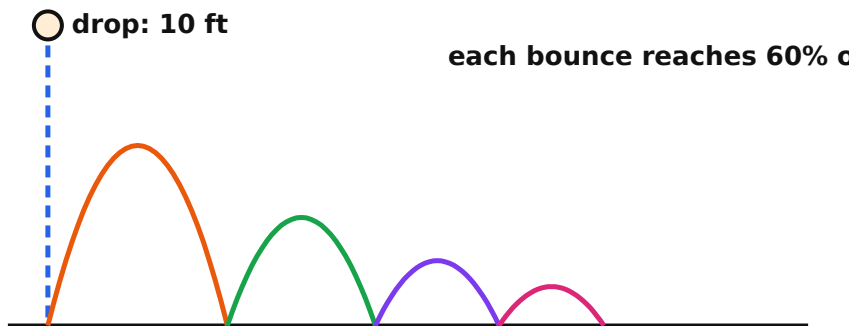
$$\frac{4}{11}$$

Exercise 19 - Salary

$$a_{10} = 42000 + 2000(9) = 60000. \quad S_{10} = \frac{10}{2}(42000 + 60000) = 510,000.$$

\$60,000; \$510,000

Exercise 20 - Bouncing ball



Each bounce is up and down.

The first bounce reaches 6 ft, and the heights are 6, 3.6, 2.16, ... with ratio 0.6. Sum of the up-heights: $\frac{6}{1-0.6} = 15$. Each is traveled up and down: $2 \cdot 15 = 30$. Add the first drop: $10 + 30$.

40 ft

Exercise 21 - Doubling pennies

Geometric with $a_1 = 1$, $r = 2$, $n = 20$: $S_{20} = \frac{2^{20} - 1}{2 - 1} = 1,048,575$ cents = \$10,485.75.

\$10,485.75