

Precalculus - Transformations of Graphs - Lesson

Common Core F-BF.B.3, F-IF.C.7b · maths-pdf.fr/en/

Transformations move or reshape a parent graph. In $y = a f(b(x - h)) + k$, h shifts right, k shifts up, a stretches vertically and reflects over the x -axis when negative, and b compresses horizontally. Inside changes act opposite to their sign, so $f(x - 3)$ shifts the graph right 3 units.

Precalculus · Common Core State Standards: **F-BF.B.3, F-IF.C.7b**

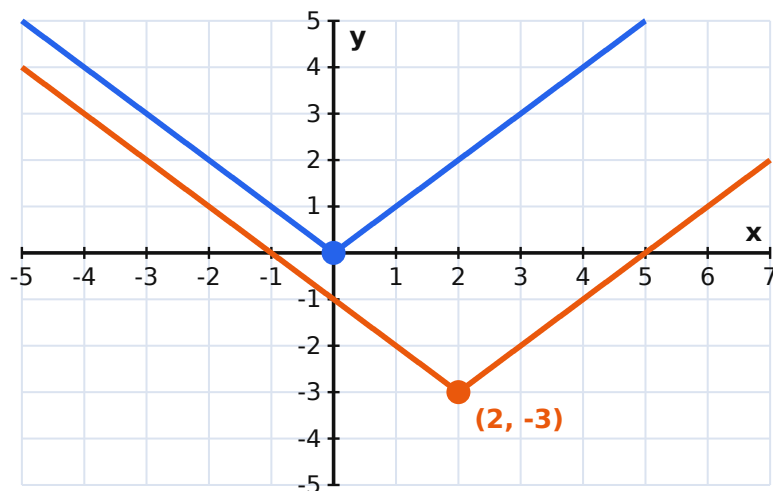
Key facts

- $f(x) + k$ shifts up k ; $f(x - h)$ shifts right h .
- $-f(x)$ reflects over the x -axis; $f(-x)$ reflects over the y -axis.
- $a f(x)$ stretches vertically by $|a|$; $f(bx)$ compresses horizontally by $1/|b|$.
- A point (p, q) on f becomes $(p + h, a q + k)$ on $a f(x - h) + k$.
- Even functions satisfy $f(-x) = f(x)$; odd functions satisfy $f(-x) = -f(x)$.

What are the basic transformations of a graph?

Start from a parent function such as $f(x) = x^2$, $|x|$, $\sqrt{x}$, x^3 or $\frac{1}{x}$. The graph of $y = a f(b(x - h)) + k$ is the parent graph moved and reshaped. Changes outside f act on y the way you expect. Changes inside the parentheses act on x in the opposite way from what the sign suggests.

Equation	Effect on the graph
$y = f(x) + k$	Shift up k units (down if $k < 0$)
$y = f(x - h)$	Shift right h units (left if $h < 0$)
$y = -f(x)$	Reflect over the x -axis
$y = f(-x)$	Reflect over the y -axis
$y = a f(x)$	Vertical stretch if $ a > 1$, compression if $0 < a < 1$
$y = f(bx)$	Horizontal compression by $\frac{1}{b}$ if $ b > 1$, stretch if $0 < b < 1$

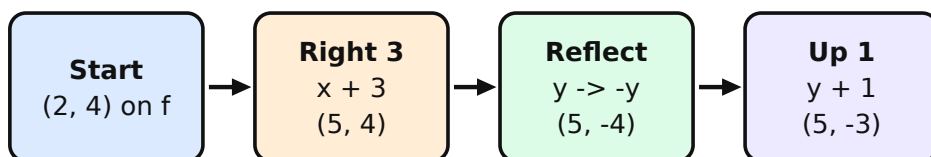


The graph of $y = |x - 2| - 3$ is the graph of $y = |x|$ moved right 2 and down 3.

How do you move a point through several transformations?

Apply the changes to each coordinate separately. A point (p, q) on f moves to $(p + h, aq + k)$ on $y = af(x - h) + k$. Use the order below when more than one change is present.

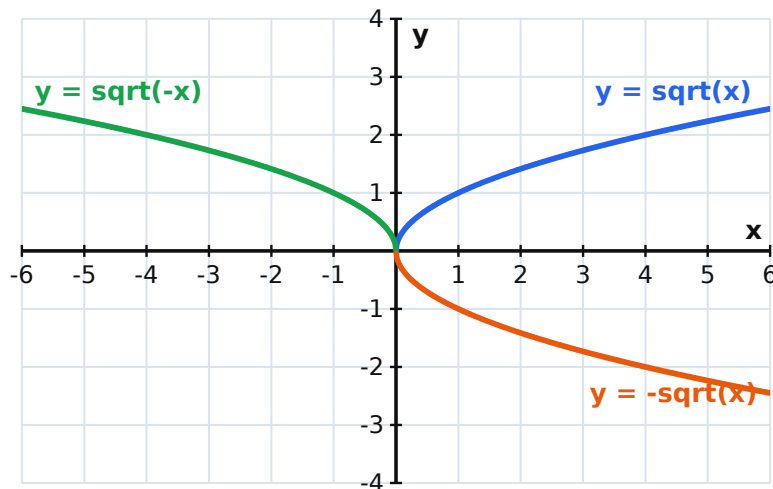
1. Horizontal changes first: stretch or compress, reflect over the y -axis, then shift left or right.
2. Vertical changes next: stretch or compress, reflect over the x -axis, then shift up or down.



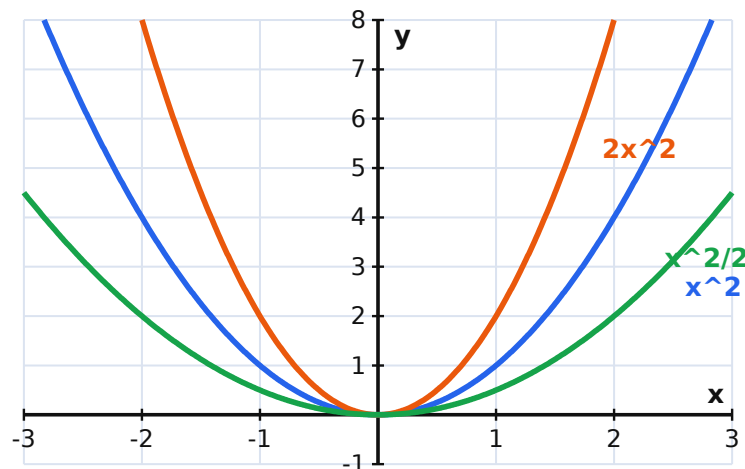
Follow one point through three moves.

How do reflections and stretches change a graph?

The sign of a controls the reflection, and $|a|$ controls the stretch. A negative inside the function flips left to right, while a negative outside flips top to bottom.



Three versions of the square root graph: reflected over the x -axis and over the y -axis.



Multiplying by 2 makes the parabola narrower; multiplying by $\frac{1}{2}$ makes it wider.

What are even and odd functions?

A function is **even** if $f(-x) = f(x)$ for all x (symmetric about the y -axis), and **odd** if $f(-x) = -f(x)$ (symmetric about the origin). For example x^2 is even, x^3 is odd, and $x^2 + x$ is neither.

Questions about transformations of graphs

Why does $f(x - 3)$ shift right instead of left?

The output $f(x - 3)$ at $x = 5$ equals the old output at $x = 2$, so every feature moves 3 units to the right. The input must be 3 larger to reach the same value.

What is the difference between $f(-x)$ and $-f(x)$?

$f(-x)$ reflects the graph over the y-axis. $-f(x)$ reflects it over the x-axis. They are equal only for special functions.

In what order do you apply transformations?

Do horizontal changes first (stretch, reflect, shift) and vertical changes next. For $y = -2\sqrt{x - 3} + 1$, shift right 3, stretch by 2, reflect over the x-axis, then shift up 1.

How can you tell if a function is even or odd?

Compute $f(-x)$. If it equals $f(x)$ the function is even. If it equals $-f(x)$ the function is odd. If neither holds, the function is neither.

How do transformations change the domain and range?

Horizontal moves shift the domain and vertical moves shift the range. Reflections and stretches can flip or scale them. The square root shifted left 4 has domain $x \geq -4$.