

Precalculus - Transformations of Graphs - Answer Key

Common Core F-BF.B.3, F-IF.C.7b · maths-pdf.fr/en/

Use this key to check your work. Each solution names the transformation or shows the substitution, and final answers are in bold.

Precalculus · Common Core State Standards: **F-BF.B.3, F-IF.C.7b**

Exercise 1 - Read a horizontal shift

$$x+3 = x - (-3), \text{ so } h = -3.$$

Shifted left 3 units

Exercise 2 - Read a vertical shift

The -5 is added to the output.

Shifted down 5 units

Exercise 3 - Reflection

The negative sign is outside the function, so it changes the sign of y .

Reflected over the x -axis

Exercise 4 - Vertical stretch

Every output is multiplied by 2.

Vertical stretch by a factor of 2

Exercise 5 - Move a point

Add 4 to x and 1 to y : $(2+4, 5+1)$.

(6,6)

Exercise 6 - Reflect and shift a point

$$x \rightarrow 3-1=2. \quad y \rightarrow -(-2)+3=5.$$

(2,5)

Exercise 7 - Write an equation

Reflect: $y = -x^2$. Shift right 3: $y = -(x-3)^2$. Shift down 2.

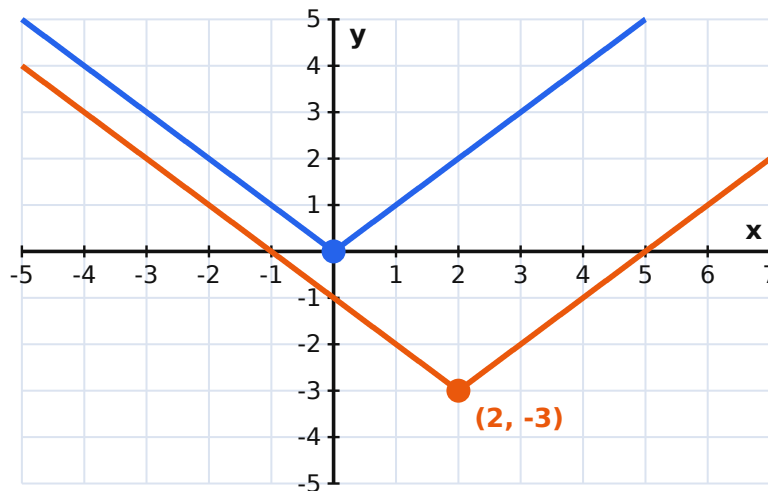
$$y = -(x-3)^2 - 2$$

Exercise 8 - Vertex of a transformed parabola

In $a(x-h)^2+k$, $h=-1$ and $k=-6$. Since $a=2>0$, it opens upward.

Vertex $(-1,-6)$; **opens up**

Exercise 9 - Identify from a graph



The vertex moved from (0, 0) to (2, -3).

The vertex is at $(2,-3)$, so shift right 2 and down 3.

$$y = |x-2|-3$$

Exercise 10 - Horizontal changes of a point

For $f(2x)$, the x -values are halved: $(2,6)$. For $f(x/3)$, the x -values are tripled: $(9,-1)$.

$(2,6)$ and $(9,-1)$

Exercise 11 - Even or odd?

(a) $f(-x) = x^4 - 3x^2 = f(x)$: even. (b) $g(-x) = -x^3 - x = -g(x)$: odd. (c) $h(-x) = x^2 - x$, which is neither $h(x)$ nor $-h(x)$.

(a) even; (b) odd; (c) neither

Exercise 12 - Sequence of transformations

Shift right 2, stretch vertically by 3, reflect over the x -axis, shift up 1. At $x=6$:
 $-3\sqrt{4}+1 = -6+1 = -5$.

Value -5

Exercise 13 - Domain and range after shifts

$x+4 \geq 0$, so $x \geq -4$. The root is at least 0, so $y \geq -3$.

Domain $[-4, \infty)$; range $[-3, \infty)$

Exercise 14 - Transform a table

x	0	2	4	6
g(x)	2	4	0	3

Values of g .

Add 2 to each x and 1 to each y .

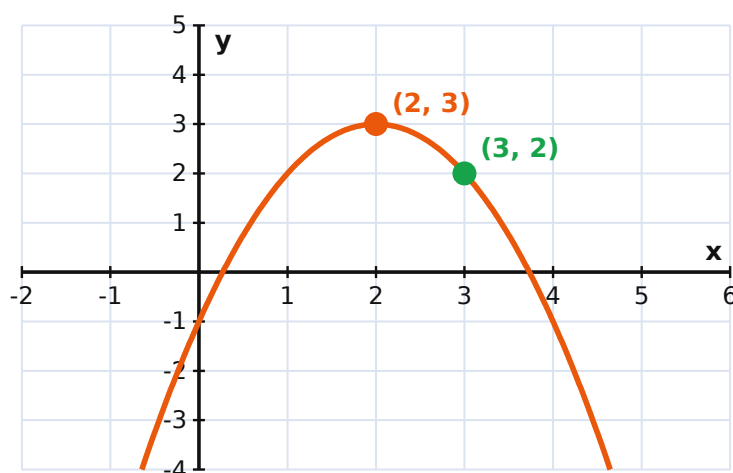
See the table

Exercise 15 - Reflect over the y-axis

Need $-x \geq 0$, so $x \leq 0$. Reflect over the y-axis: $(-9, 3)$.

Domain $(-\infty, 0]$; **point** $(-9, 3)$

Exercise 16 - Equation from a graph

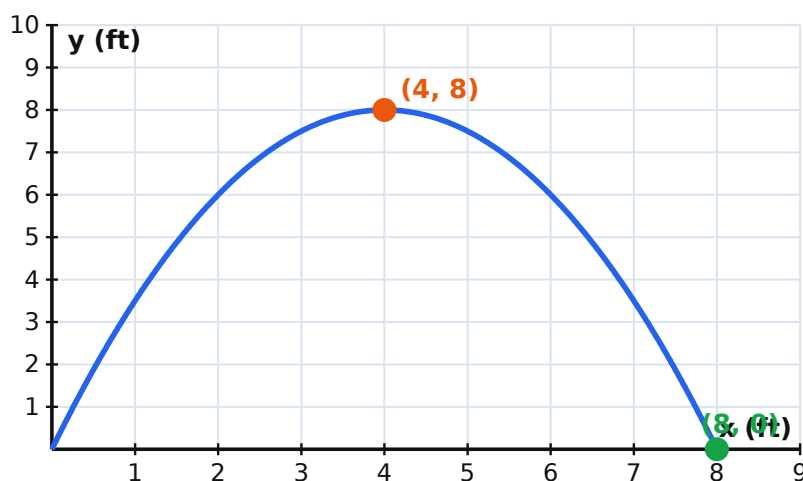


Vertex and one point.

$h = 2$, $k = 3$. Use $(3, 2)$: $2 = a(1)^2 + 3$, so $a = -1$.

$$y = -(x - 2)^2 + 3$$

Exercise 17 - Fountain



Top at $(4, 8)$; landing at $(8, 0)$.

Maximum: $k=8$ ft at $x=4$. Set $y=0$: $(x-4)^2=16$, so $x=0$ or $x=8$. The graph is $y=x^2$ reflected over the x -axis, compressed by $\frac{1}{2}$, shifted right 4 and up 8.

8 ft; lands at $x=8$ ft

Exercise 18 - Taxi fare transformation

$N(m)=0.8f(m)+4$: vertical compression by 0.8, then shift up 4. $N(10)=0.8(25)+4=24$.

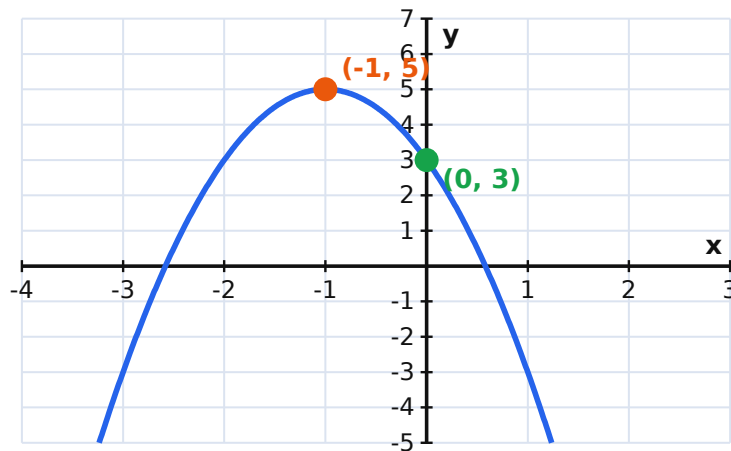
Compress by 0.8, shift up 4; $N(10)=\$24$

Exercise 19 - Who is right?

$|3x+2|=|3(x+\frac{2}{3})|$, so it shifts left $\frac{2}{3}$, not 2. For a shift left 2 after compressing, use $y=|3(x+2)|=|3x+6|$.

No; the correct equation is $y=|3(x+2)|$

Exercise 20 - All features of a transformed parabola



Graph of g .

Vertex $(-1,5)$; since $a<0$, the range is $(-\infty,5]$. $g(0)=-2+5=3$. Solve $-2(x+1)^2+5=0$: $(x+1)^2=2.5$, $x=-1\pm\sqrt{2.5}\approx 0.58, -2.58$.

Vertex $(-1,5)$; range $(-\infty,5]$; y -int 3; x -int about 0.58 and -2.58