

Precalculus - Graphs of Trigonometric Functions - Lesson

Common Core F-TF.B.5, F-IF.C.7e, F-TF.B.7 · maths-pdf.fr/en/

Sine and cosine graphs are periodic waves between -1 and 1 that repeat every 2π . In $y = a \sin(b(x - h)) + k$, the amplitude is $|a|$, the period is $2\pi / |b|$, the phase shift is h and the midline is $y = k$. Tangent has period π and vertical asymptotes where cosine is zero.

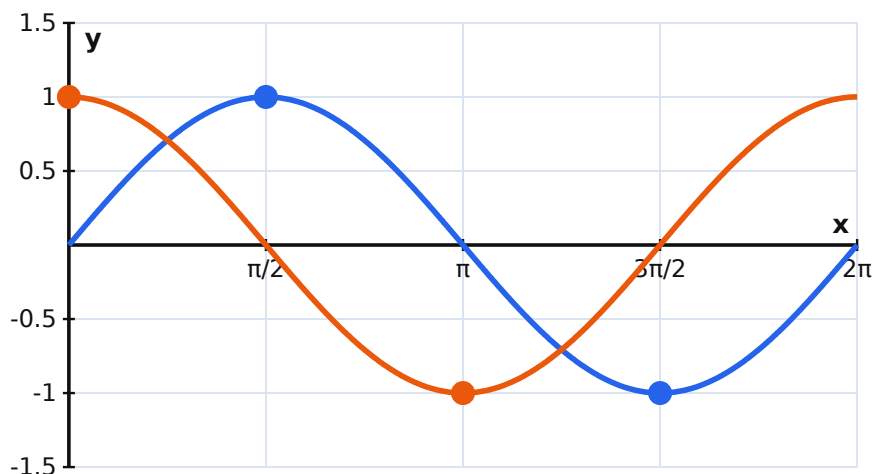
Precalculus · Common Core State Standards: **F-TF.B.5, F-IF.C.7e, F-TF.B.7**

Key facts

- Amplitude = $|a| = (\max - \min)/2$; midline $y = k = (\max + \min)/2$.
- Period of sine and cosine = $2\pi / |b|$; period of tangent = $\pi / |b|$.
- Phase shift h moves the graph right when h is positive.
- Tangent has vertical asymptotes at $x = \pi/2 + k\pi$.
- Use cosine for a model that starts at a maximum and sine for one that starts at the midline.

What do the graphs of sine and cosine look like?

The sine and cosine functions are **periodic**: their graphs repeat every 2π . Both oscillate between -1 and 1 with a smooth wave. The cosine graph starts at its maximum $(0,1)$; the sine graph starts at the midline $(0,0)$ and rises. The cosine graph is the sine graph shifted left $\frac{\pi}{2}$.

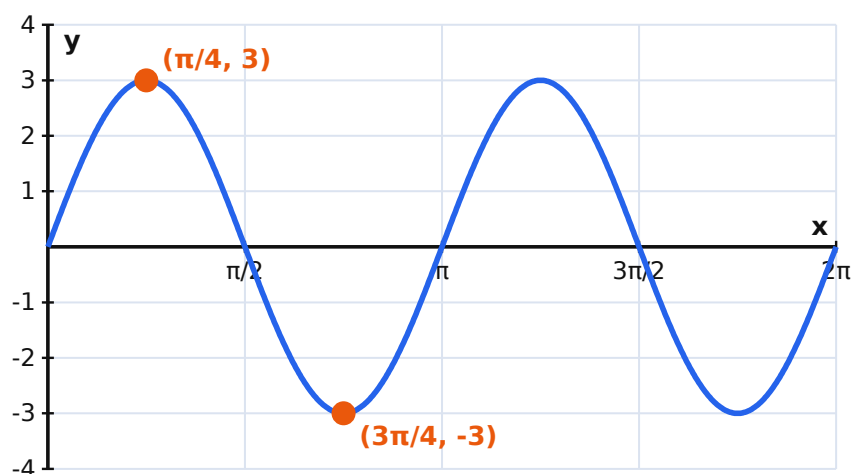


One period of sine (blue) and cosine (orange).

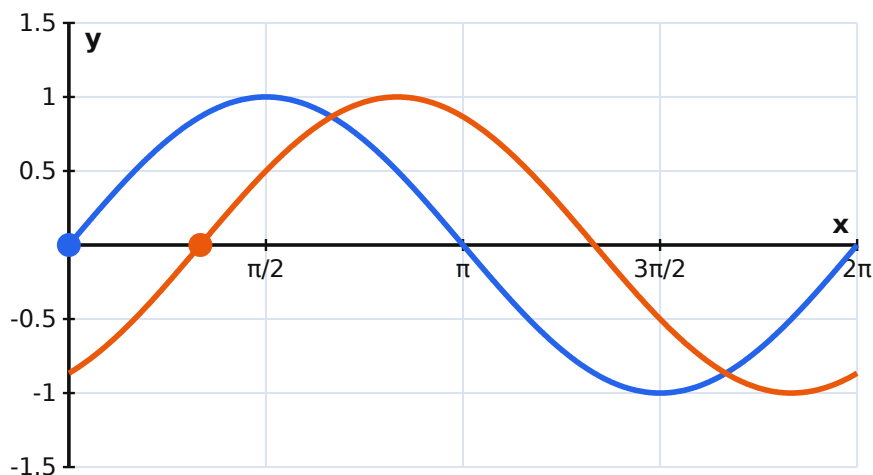
How do you read the equation $y = a\sin(b(x-h)) + k$?

Constant	Meaning	Formula
$ a $	Amplitude (distance from midline to peak)	$\frac{\max - \min}{2}$
b	Controls the period	$\text{period} = \frac{2\pi}{ b }$
h	Phase shift (right if positive)	Start of a cycle
k	Vertical shift and midline $y = k$	$\frac{\max + \min}{2}$

A negative a reflects the graph over the midline. For $y = 3\sin 2x$, the amplitude is 3 and the period is $\frac{2\pi}{2} = \pi$.



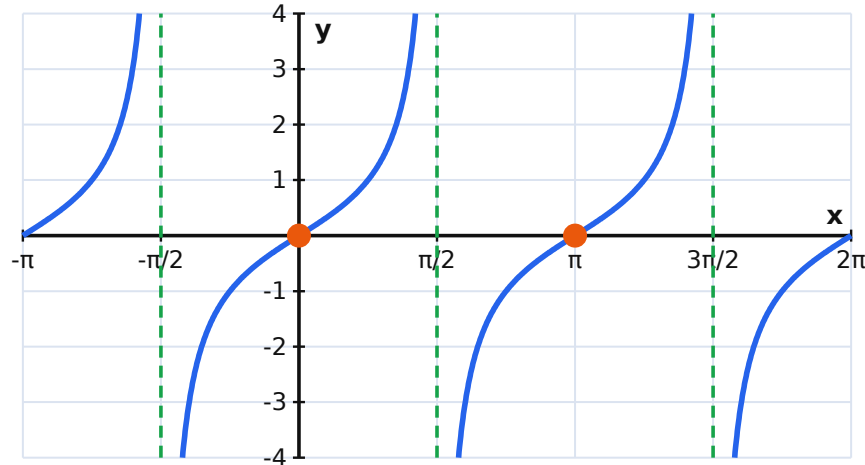
$y = 3\sin 2x$: maximum 3 at $x = \frac{\pi}{4}$, period π .



$y = \sin\left(x - \frac{\pi}{3}\right)$ is the sine graph shifted right $\frac{\pi}{3}$.

What does the tangent graph look like?

Since $\tan x = \frac{\sin x}{\cos x}$, tangent is undefined where $\cos x = 0$, at $x = \frac{\pi}{2} + k\pi$. These are vertical asymptotes. The period of tangent is π , the graph passes through the origin, and it increases without bound between asymptotes. For $y = \tan bx$ the period is $\frac{\pi}{|b|}$.



Tangent repeats every π and has vertical asymptotes at odd multiples of $\pi/2$.

How do you build a trigonometric model?

1. Find the midline $k = \frac{\max + \min}{2}$ and amplitude $|a| = \frac{\max - \min}{2}$.
2. Find the period P and set $b = \frac{2\pi}{P}$.
3. Choose cosine if the model starts at a maximum (or sine if it starts at the midline) to avoid a phase shift.
4. Check one or two points.

For tides, a depth that ranges from 3 ft to 7 ft with a 12-hour cycle and a high tide at $t = 0$ is $d(t) = 5 + 2\cos\left(\frac{\pi}{6}t\right)$, because the amplitude is 2, the midline is 5, and $b = \frac{2\pi}{12} = \frac{\pi}{6}$.



Questions about graphs of trigonometric functions

What is amplitude?

Amplitude is the distance from the midline to a maximum or minimum. For $y = 3 \sin x + 2$ the amplitude is 3, and the graph goes from -1 to 5.

How do you find the period?

Divide 2π by the absolute value of b for sine and cosine, or π by the absolute value of b for tangent. For $y = \sin(4x)$, the period is $2\pi / 4 = \pi / 2$.

What is a phase shift?

It is a horizontal shift. Write the function as $\sin(b(x - h))$; then h is the shift. For $\sin(2x - \pi) = \sin(2(x - \pi/2))$ the shift is $\pi/2$ to the right.

Why does tangent have asymptotes?

Tangent is sine divided by cosine. Where cosine is 0, the quotient is undefined and the graph shoots off to infinity, creating a vertical asymptote.

How do I choose sine or cosine for a model?

Either works with a phase shift. Cosine is simpler if the data starts at a maximum, and sine is simpler if it starts at the midline going up.