

# Precalculus - Graphs of Trigonometric Functions - Answer Key

Common Core F-TF.B.5, F-IF.C.7e, F-TF.B.7 · [maths-pdf.fr/en/](https://maths-pdf.fr/en/)

Use this key to check each trig graph or model. Solutions show the amplitude, period and midline calculations, with final answers in bold.

**Precalculus** · Common Core State Standards: **F-TF.B.5, F-IF.C.7e, F-TF.B.7**

## Exercise 1 - Amplitude and period

Amplitude  $|3|=3$ . Period  $\frac{2\pi}{\frac{1}{3}} = \pi$ .

**Amplitude 3; period  $\pi$**

## Exercise 2 - Reflected cosine

Amplitude  $\frac{1}{2}$ . Here  $b = \frac{1}{3}$ , so the period is  $\frac{2\pi}{1/3} = 6\pi$ . The negative sign reflects the graph over the  $x$ -axis.

**Amplitude  $\frac{1}{2}$ ; period  $6\pi$ ; reflected**

## Exercise 3 - Period with pi in b

$$\frac{2\pi}{\pi} = 2.$$

**2**

## Exercise 4 - Phase shifts

(a)  $\frac{\pi}{3}$  to the right. (b)  $\cos(2x + \pi) = \cos\left(2\left(x + \frac{\pi}{2}\right)\right)$ : shift  $\frac{\pi}{2}$  left.

**(a) right  $\frac{\pi}{3}$ ; (b) left  $\frac{\pi}{2}$**

## Exercise 5 - Midline and range

Midline  $y = -3$ . Maximum  $-3+2 = -1$ , minimum  $-3-2 = -5$ .

**Midline**  $y = -3$ ; **range**  $[-5, -1]$

### Exercise 6 - Write an equation

$$b = \frac{2\pi}{\pi} = 2.$$

$$y = 2\sin 2x + 1$$

### Exercise 7 - Evaluate

$$3\sin\frac{\pi}{2} = 3. \quad 3\sin\frac{\pi}{3} = \frac{3\sqrt{3}}{2}.$$

$$3 \text{ and } \frac{3\sqrt{3}}{2}$$

### Exercise 8 - Table of values

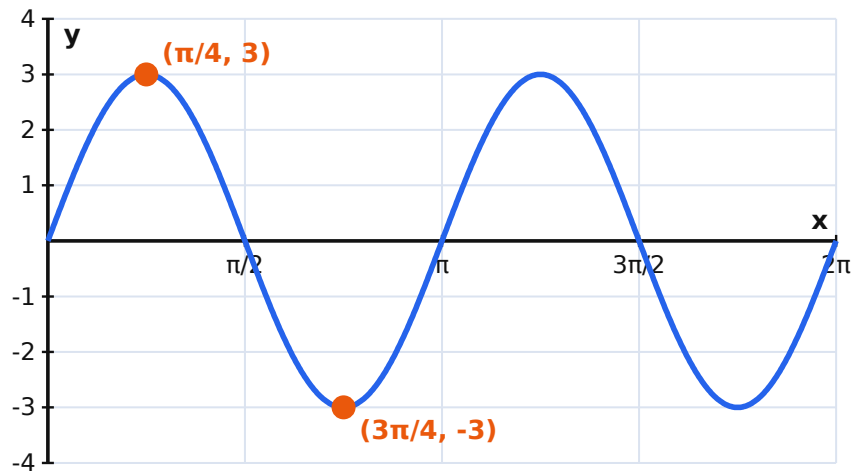
| x         | 0 | $\pi/3$ | $\pi/2$ | $\pi$ |
|-----------|---|---------|---------|-------|
| $2\cos x$ | 2 | 1       | 0       | -2    |

Completed table.

$$2\cos 0 = 2, \quad 2\cos\frac{\pi}{3} = 1, \quad 2\cos\frac{\pi}{2} = 0, \quad 2\cos\pi = -2.$$

$$2, 1, 0, -2$$

### Exercise 9 - Read a graph



First maximum at  $(\pi/4, 3)$ .

The peak height is 3. The curve completes a cycle in  $\pi$ . The first maximum occurs a quarter period in:  $\frac{\pi}{4}$ .

**Amplitude 3; period  $\pi$ ;  $(\frac{\pi}{4}, 3)$**

### Exercise 10 - Tangent with a stretch

Period  $\frac{\pi}{1/2} = 2\pi$ . Asymptotes where  $\frac{x}{2} = \pm \frac{\pi}{2}$ :  $x = \pm \pi$ .

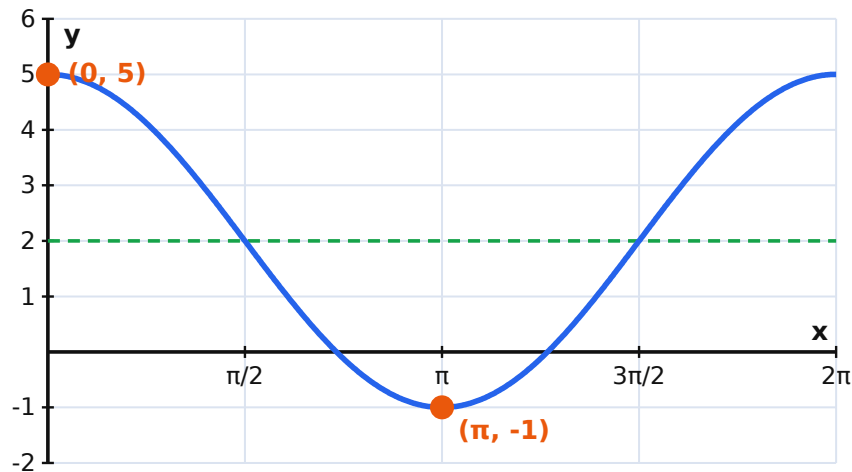
**Period  $2\pi$ ; asymptotes  $x = -\pi, \pi$**

### Exercise 11 - Tangent with compression

Period  $\frac{\pi}{2}$ . The first asymptote is where  $2x = \frac{\pi}{2}$ .

**Period  $\frac{\pi}{2}$ ;  $x = \frac{\pi}{4}$**

### Exercise 12 - Equation from a graph (cosine)

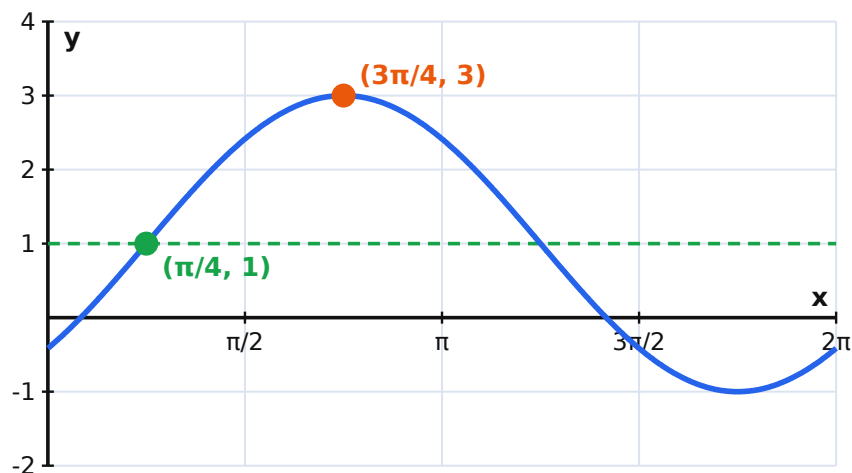


Max 5 at 0, min -1 at  $\pi$ .

Midline  $k = \frac{5+(-1)}{2} = 2$ . Amplitude  $\frac{5-(-1)}{2} = 3$ . The half period is  $\pi$ , so the period is  $2\pi$  and  $b=1$ .

$$y = 3\cos x + 2$$

### Exercise 13 - Equation from a graph (shifted sine)



Rises through the midline at  $\pi/4$ .

A sine curve rises through the midline at the start of a cycle, so the phase shift is  $\frac{\pi}{4}$  to the right. Check: the maximum is a quarter period later,  $\frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$ .

$$y = 2\sin\left(x - \frac{\pi}{4}\right) + 1$$

### Exercise 14 - Range

$\cos x$  ranges over  $[-1, 1]$ , so  $-4\cos x$  ranges over  $[-4, 4]$ . Add 1.

$[-3, 5]$

### Exercise 15 - Phase shift identity

$$\sin \frac{\pi}{2} = 1 = \cos 0. \quad \sin \frac{5\pi}{6} = \frac{1}{2} = \cos \frac{\pi}{3}. \quad \sin \frac{3\pi}{2} = -1 = \cos \pi.$$

**Shifting sine left  $\pi/2$  gives cosine**

### Exercise 16 - Alternating current

Amplitude 170. Period  $\frac{2\pi}{120\pi} = \frac{1}{60}$  s. Frequency = 60 cycles per second.

**170 V;  $\frac{1}{60}$  s; 60 cycles per second**

### Exercise 17 - Tide



High 7 ft at  $t = 0$ ; low 3 ft at  $t = 6$ .

Maximum  $5+2=7$ , minimum  $5-2=3$ . Period  $\frac{2\pi}{\pi/6} = 12$  h.  $d(2) = 5 + 2\cos \frac{\pi}{3} = 5 + 1 = 6$ .

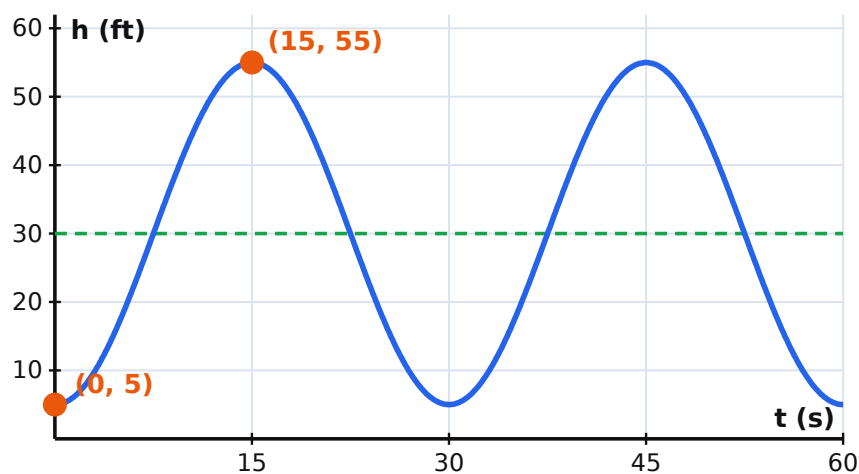
**7 ft; 3 ft; 12 h; 6 ft**

### Exercise 18 - Daylight hours

Amplitude 2.5. The maximum is  $12+2.5=14.5$  hours.  $D(80) = 12 + 2.5\sin 0 = 12$ .

2.5; 14.5 h; 12 h

### Exercise 19 - Ferris wheel



Low 5 ft at  $t = 0$ ; high 55 ft at  $t = 15$ .

Lowest  $30 - 25 = 5$  ft, highest 55 ft. Period  $\frac{2\pi}{\pi/15} = 30$  s.  $h(10) = 30 - 25\cos\frac{2\pi}{3} = 30 + 12.5 = 42.5$ .

5 ft; 55 ft; 30 s; 42.5 ft

### Exercise 20 - A model from data

Midline 3, amplitude 4. Half period  $4\pi$ , so period  $8\pi$  and  $b = \frac{2\pi}{8\pi} = \frac{1}{4}$ . Check: at  $x = 4\pi$ ,  $4\cos\pi + 3 = -1$ .

$$y = 4\cos\frac{x}{4} + 3$$