

Precalculus - Trigonometric Identities - Lesson

Common Core F-TF.C.8, F-TF.C.9 · maths-pdf.fr/en/

Trigonometric identities are equations true for every allowed angle. The Pythagorean identity $\sin^2 x + \cos^2 x = 1$ comes from the unit circle. Sum, difference and double-angle formulas, such as $\sin 2A = 2 \sin A \cos A$, let you find exact values, simplify expressions and solve equations.

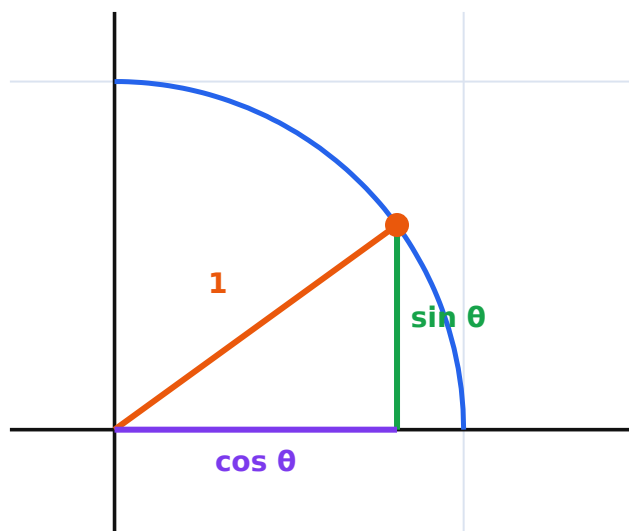
Precalculus · Common Core State Standards: **F-TF.C.8, F-TF.C.9**

Key facts

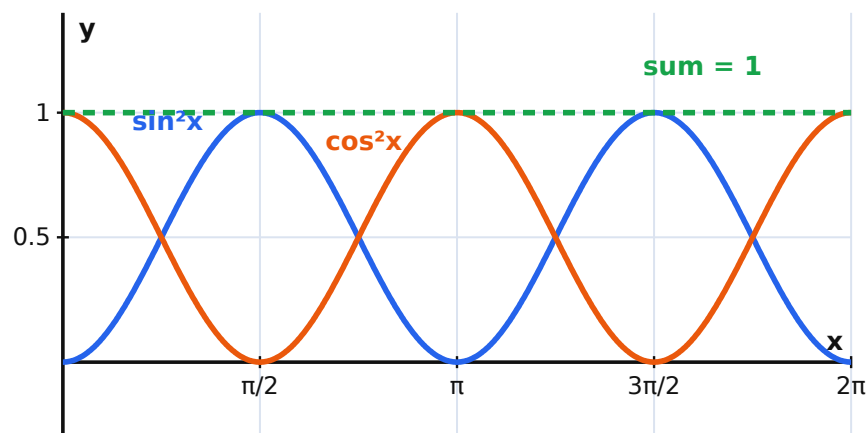
- $\sin^2 x + \cos^2 x = 1$ and $1 + \tan^2 x = 1/\cos^2 x$.
- $\tan x = \sin x / \cos x$; csc, sec, cot are reciprocals of sin, cos, tan.
- $\sin(A + B) = \sin A \cos B + \cos A \sin B$; $\cos(A + B) = \cos A \cos B - \sin A \sin B$.
- $\sin 2A = 2 \sin A \cos A$; $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1$.
- To verify an identity, work on one side only until it matches the other.

What is a trigonometric identity?

An identity is an equation that is true for every value where both sides are defined. The basic ones come from the unit circle. A point at angle θ is $(\cos\theta, \sin\theta)$ and lies one unit from the origin, so $\sin^2\theta + \cos^2\theta = 1$. Dividing by $\cos^2\theta$ gives $1 + \tan^2\theta = \frac{1}{\cos^2\theta}$. The quotient identity is $\tan\theta = \frac{\sin\theta}{\cos\theta}$, and the reciprocal functions are $\csc\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$.



In the unit circle the legs of the right triangle are $\cos\theta$ and $\sin\theta$, and the hypotenuse is 1.

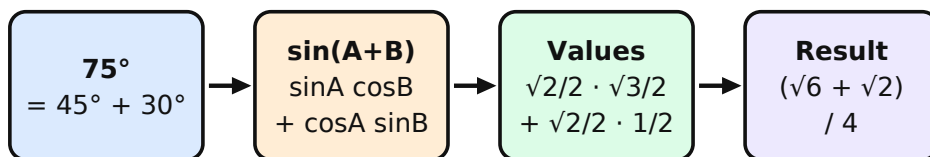


The squares of sine and cosine always add up to 1.

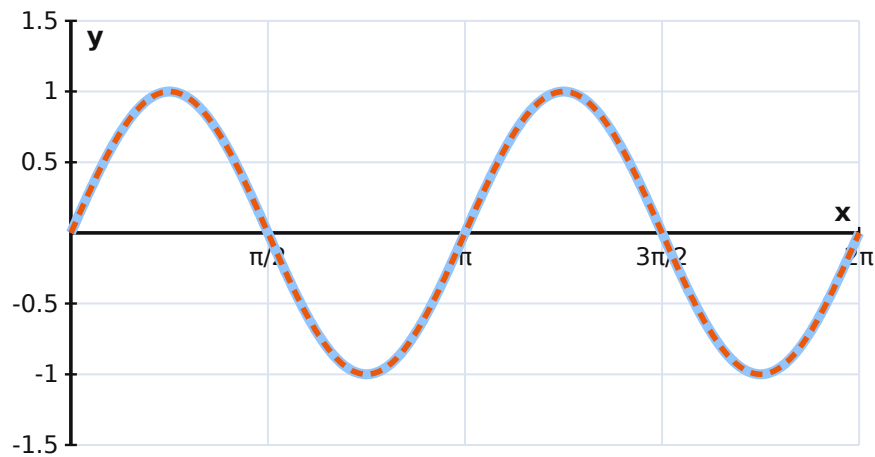
What are the sum, difference and double-angle formulas?

Formula	Identity
Sine of a sum	$\sin(A+B) = \sin A \cos B + \cos A \sin B$
Cosine of a sum	$\cos(A+B) = \cos A \cos B - \sin A \sin B$
Sine of a difference	$\sin(A-B) = \sin A \cos B - \cos A \sin B$
Cosine of a difference	$\cos(A-B) = \cos A \cos B + \sin A \sin B$
Double angle	$\sin 2A = 2 \sin A \cos A$; $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1$

Example: Find $\sin 75^\circ$. Write $75^\circ = 45^\circ + 30^\circ$ and use the sum formula.



$$\sin 75^\circ = \frac{\sqrt{6} + \sqrt{2}}{4} \approx 0.966.$$



The graphs of $\sin 2x$ and $2\sin x \cos x$ coincide, which illustrates the double-angle formula.

Why do the sum formulas matter? The special angles on the unit circle give exact values only for multiples of 30° and 45° . Adding or subtracting those angles reaches many more, such as 15° , 75° and 105° , with exact answers that contain square roots. The double-angle formulas go the other way: they express a function of 2θ using functions of θ . If you know $\sin \theta$ and the quadrant, you can find $\cos \theta$ from the Pythagorean identity and then every double-angle value, without ever finding θ itself.

How do you verify an identity?

1. Start with the more complicated side.
2. Rewrite in terms of sine and cosine, use a common denominator, or apply an identity.
3. Simplify until it matches the other side. Never move terms across the equals sign.

Identities also help solve equations. To solve $\sin 2x = \sin x$, rewrite $2\sin x \cos x - \sin x = 0$, factor $\sin x(2\cos x - 1) = 0$, and solve each factor. The half-angle forms $\sin^2 x = \frac{1 - \cos 2x}{2}$ and $\cos^2 x = \frac{1 + \cos 2x}{2}$ reduce powers.

Questions about trigonometric identities

What is the difference between an identity and an equation?

An identity is true for all values where it is defined, like $\sin^2 x + \cos^2 x = 1$. A conditional equation, like $\sin x = 1/2$, is true only for certain values.

How do you verify a trig identity?

Pick one side, usually the more complicated, and transform it with known identities until it equals the other side. Do not treat it like an equation by adding to both sides.

Why is $\sin(A + B)$ not $\sin A + \sin B$?

Sine is not a linear function. For $A = B = 90$ degrees, $\sin(A + B) = \sin 180 = 0$, but $\sin A + \sin B = 2$.

Which form of $\cos 2A$ should I use?

Choose the form that matches what you know. Use $1 - 2 \sin^2 A$ if you know sine, $2 \cos^2 A - 1$ if you know cosine, or $\cos^2 A - \sin^2 A$ if you know both.

How can I find exact values like $\sin 75$ degrees?

Write the angle as a sum or difference of special angles, such as $45 + 30$, then apply the sum or difference formula.