

Precalculus - Trigonometric Identities - Answer Key

Common Core F-TF.C.8, F-TF.C.9 · maths-pdf.fr/en/

Use this key to check each identity or exact value. Verification solutions work on one side only, and final answers are in bold.

Precalculus · Common Core State Standards: **F-TF.C.8, F-TF.C.9**

Exercise 1 - Simplify a product

$$\frac{\sin x}{\cos x} \cdot \cos x = \sin x.$$

$$\sin x$$

Exercise 2 - Pythagorean simplification

$$1 - \cos^2 x = \sin^2 x, \text{ so } \frac{\sin^2 x}{\sin x}.$$

$$\sin x$$

Exercise 3 - Sum of squares

$$1 + \tan^2 x = \frac{1}{\cos^2 x}.$$

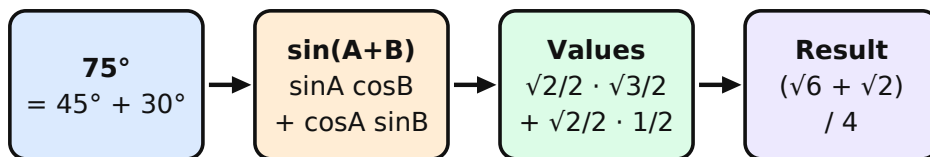
$$\frac{1}{\cos^2 x}$$

Exercise 4 - Double angle values

$$\cos \theta = -\frac{4}{5}, \sin 2\theta = 2 \cdot \frac{3}{5} \cdot \left(-\frac{4}{5}\right) = -\frac{24}{25}, \cos 2\theta = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}.$$

$$\sin 2\theta = -\frac{24}{25}, \cos 2\theta = \frac{7}{25}$$

Exercise 5 - Exact sine of 15 degrees



Use the difference formula this time.

$$\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}.$$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

Exercise 6 - Exact cosine of 15 degrees

$$\cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}.$$

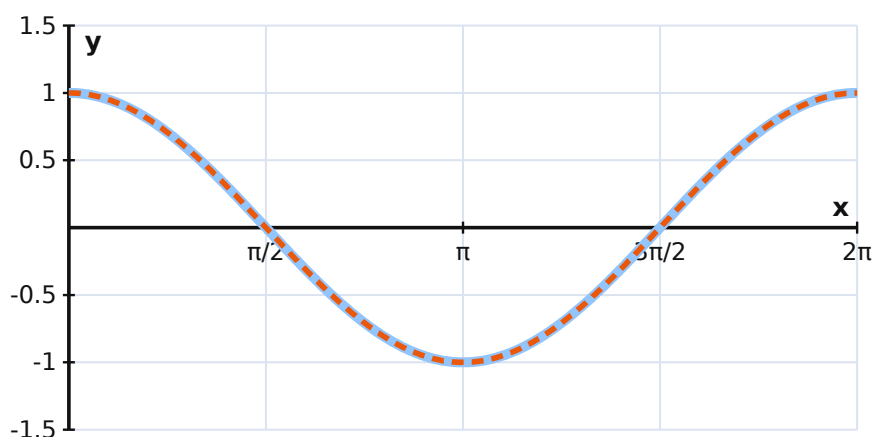
$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

Exercise 7 - Tangent sum

$$\frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}}. \text{ Multiply by } \frac{3 + \sqrt{3}}{3 + \sqrt{3}}: \frac{12 + 6\sqrt{3}}{6} = 2 + \sqrt{3}.$$

$$2 + \sqrt{3}$$

Exercise 8 - Shift identity



The curves coincide.

$$\sin x \cos \frac{\pi}{2} + \cos x \sin \frac{\pi}{2} = \sin x \cdot 0 + \cos x \cdot 1 = \cos x.$$

The identity holds

Exercise 9 - Verify a square

$$\sin^2 x + 2\sin x \cos x + \cos^2 x = (\sin^2 x + \cos^2 x) + 2\sin x \cos x = 1 + \sin 2x.$$

Verified

Exercise 10 - Verify with fractions

Common denominator $\sin x(1 + \cos x)$: numerator

$$\sin^2 x + (1 + \cos x)^2 = \sin^2 x + 1 + 2\cos x + \cos^2 x = 2 + 2\cos x = 2(1 + \cos x). \text{ Cancel } 1 + \cos x.$$

Verified

Exercise 11 - Verify tangent plus cotangent

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x}.$$

Verified

Exercise 12 - Half-angle power

$$\cos 2\pi/8 = \frac{1 + \cos \pi/4}{2} = \frac{1 + \frac{\sqrt{2}}{2}}{2} = \frac{2 + \sqrt{2}}{4}.$$

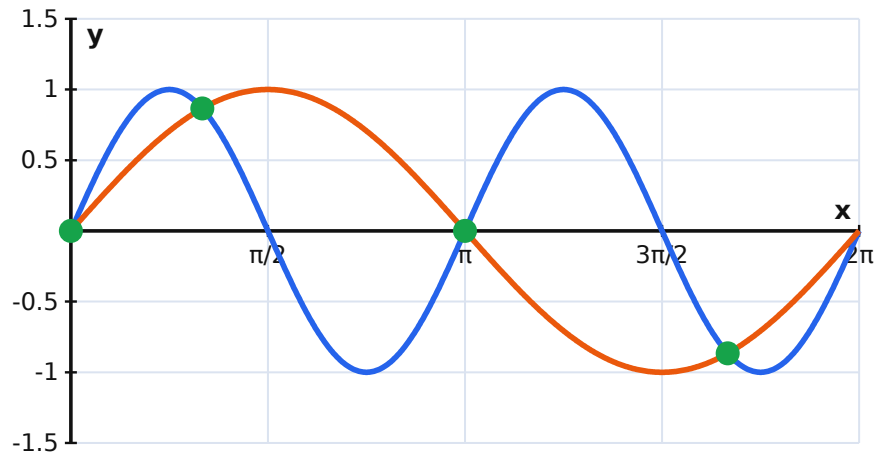
$$\frac{2 + \sqrt{2}}{4}$$

Exercise 13 - Cosine double angle from sine

$$\cos 2x = 1 - 2\sin^2 x = 1 - \frac{2}{9}.$$

$$\frac{7}{9}$$

Exercise 14 - Solve with a double angle



Four intersections.

$$2\sin x \cos x - \sin x = 0, \text{ so } \sin x(2\cos x - 1) = 0. \sin x = 0: x = 0, \pi. \cos x = \frac{1}{2}: x = \frac{\pi}{3}, \frac{5\pi}{3}.$$

$$x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

Exercise 15 - Solve $\cos 2x = \cos x$

$$2\cos^2 x - 1 = \cos x, \text{ so } 2\cos^2 x - \cos x - 1 = 0 \text{ and } (2\cos x + 1)(\cos x - 1) = 0. \cos x = 1: 0. \cos x = -\frac{1}{2}: \frac{2\pi}{3}, \frac{4\pi}{3}.$$

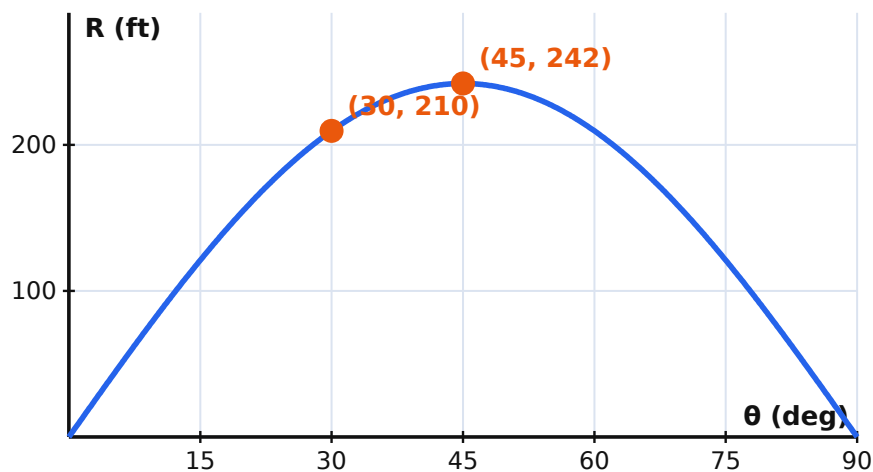
$$x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$$

Exercise 16 - Use Pythagorean to solve

$$\text{Replace } \cos^2 x = 1 - \sin^2 x: 2 - 2\sin^2 x + \sin x - 1 = 0, \text{ so } 2\sin^2 x - \sin x - 1 = 0 \text{ and } (2\sin x + 1)(\sin x - 1) = 0.$$

$$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Exercise 17 - Projectile range



Maximum range at 45 degrees.

$$R(30^\circ) = \frac{7744 \sin 60^\circ}{32} \approx 210. \quad R(45^\circ) = \frac{7744 \sin 90^\circ}{32} = 242.$$

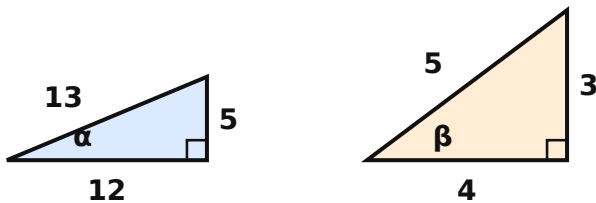
210 ft and 242 ft

Exercise 18 - Electrical amplitude

$$\sin t \cos t = \frac{1}{2} \sin 2t, \text{ so } V = 85 \sin 2t.$$

$$V = 85 \sin 2t; \text{ amplitude } 85$$

Exercise 19 - Sum of two angles



Triangles 5-12-13 and 3-4-5.

$$\cos \alpha = \frac{12}{13}, \sin \beta = \frac{3}{5}. \sin(\alpha + \beta) = \frac{5}{13} \cdot \frac{4}{5} + \frac{12}{13} \cdot \frac{3}{5} = \frac{56}{65}. \cos(\alpha + \beta) = \frac{12}{13} \cdot \frac{4}{5} - \frac{5}{13} \cdot \frac{3}{5} = \frac{33}{65}.$$

$\frac{56}{65}$ and $\frac{33}{65}$

Exercise 20 - Prove a triple angle

$\cos 2x \cos x - \sin 2x \sin x = (2\cos^2 x - 1)\cos x - 2\sin^2 x \cos x$. Replace $\sin^2 x = 1 - \cos^2 x$:
 $2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x = 4\cos^3 x - 3\cos x$.

Verified