

Precalculus - Trigonometric Identities - Test Answer Key

Common Core F-TF.C.8, F-TF.C.9 · maths-pdf.fr/en/

Worked solutions and scoring notes for the 9-question trig identities test. Use the scoring line under each solution to award partial credit.

Precalculus · Common Core State Standards: **F-TF.C.8, F-TF.C.9**

Question 1 - Simplify [2 pts]

$$\sin^2 x = 1 - \cos^2 x = (1 - \cos x)(1 + \cos x). \text{ Cancel } 1 - \cos x.$$

$$1 + \cos x$$

Scoring: 1 point for factoring, 1 point for the result.

Question 2 - Exact value [2 pts]

$$\cos(45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}.$$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

Scoring: 1 point for the formula with correct values, 1 point for the result.

Question 3 - Double angle [2 pts]

$$\sin \theta = -\frac{12}{13}. \sin 2\theta = 2\left(-\frac{12}{13}\right)\frac{5}{13} = -\frac{120}{169}. \cos 2\theta = \frac{25}{169} - \frac{144}{169} = -\frac{119}{169}.$$

$$-\frac{120}{169} \text{ and } -\frac{119}{169}$$

Scoring: 1 point each.

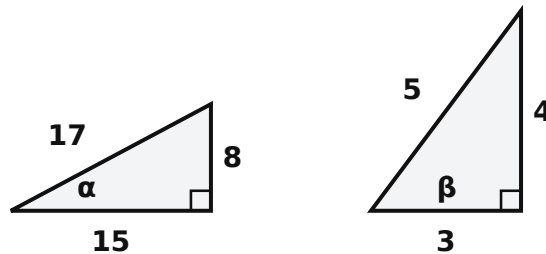
Question 4 - Verify [2 pts]

$$1 - \cos 2x = 1 - (1 - 2\sin^2 x) = 2\sin^2 x. \text{ Then } \frac{2\sin^2 x}{2\sin x} = \sin x.$$

Verified

Scoring: 1 point for the double-angle substitution, 1 point for simplifying.

Question 5 - Angle addition [3 pts]



Triangles 8-15-17 and 3-4-5.

$$\cos \alpha = \frac{15}{17}, \sin \beta = \frac{4}{5}. \text{ (a) } \frac{8}{17} \cdot \frac{3}{5} + \frac{15}{17} \cdot \frac{4}{5} = \frac{84}{85}. \text{ (b) } \frac{15}{17} \cdot \frac{3}{5} - \frac{8}{17} \cdot \frac{4}{5} = \frac{13}{85}. \text{ (c) } \frac{84/85}{13/85} = \frac{84}{13}.$$

$$\frac{84}{85}, \frac{13}{85}, \frac{84}{13}$$

Scoring: 1 point for each part.

Question 6 - Solve [2 pts]

$$2\sin x \cos x - \cos x = 0, \text{ so } \cos x(2\sin x - 1) = 0. \cos x = 0: \frac{\pi}{2}, \frac{3\pi}{2}. \sin x = \frac{1}{2}: \frac{\pi}{6}, \frac{5\pi}{6}.$$

$$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

Scoring: 1 point for factoring, 1 point for all four solutions.

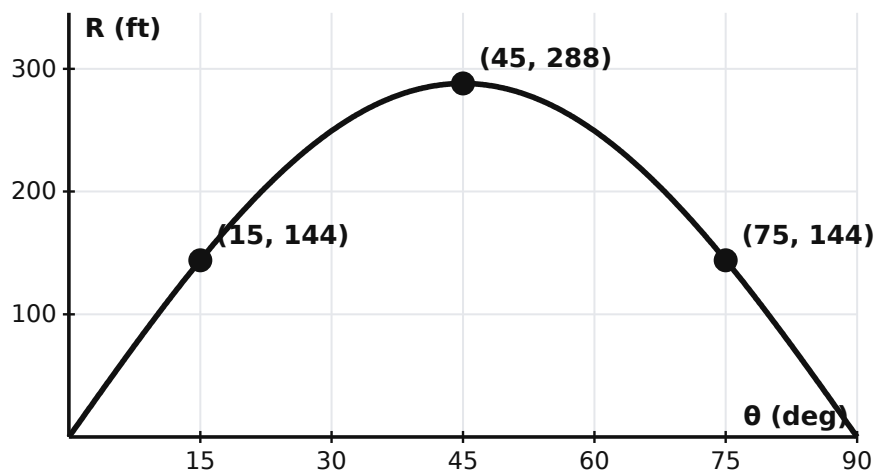
Question 7 - Solve with $\cos 2x$ [2 pts]

$$2x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \text{ so divide by 2.}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Scoring: 1 point for the values of $2x$, 1 point for the final solutions.

Question 8 - Projectile range [3 pts]



Same range at 15 and 75 degrees.

(a) $288\sin 30^\circ = 144$. (b) $288\sin 90^\circ = 288$. (c) $\sin 2\theta = \sin 30^\circ$ also holds for $2\theta = 150^\circ$, so $\theta = 75^\circ$.

144 ft; 288 ft; 75°

Scoring: 1 point for each part.

Question 9 - Verify a Pythagorean form [2 pts]

$1 + \tan^2 x = \frac{1}{\cos^2 x}$, so the product is $\frac{\cos^2 x}{\cos^2 x} = 1$.

Verified

Scoring: 1 point for the identity, 1 point for simplifying.

Total

20 points