

Precalculus - Vectors in the Plane - Lesson

Common Core N-VM.A.1, N-VM.A.2, N-VM.A.3, N-VM.B.4, N-VM.B.5 · maths-pdf.fr/en/

A vector has magnitude and direction. In component form $\langle a, b \rangle$ its magnitude is $\sqrt{a^2 + b^2}$. Add vectors and multiply by scalars component by component. The dot product $\langle a, b \rangle \cdot \langle c, d \rangle = ac + bd$ gives the angle between vectors through $\cos(\theta) = \mathbf{u} \cdot \mathbf{v} / (|\mathbf{u}||\mathbf{v}|)$, and it equals 0 for perpendicular vectors.

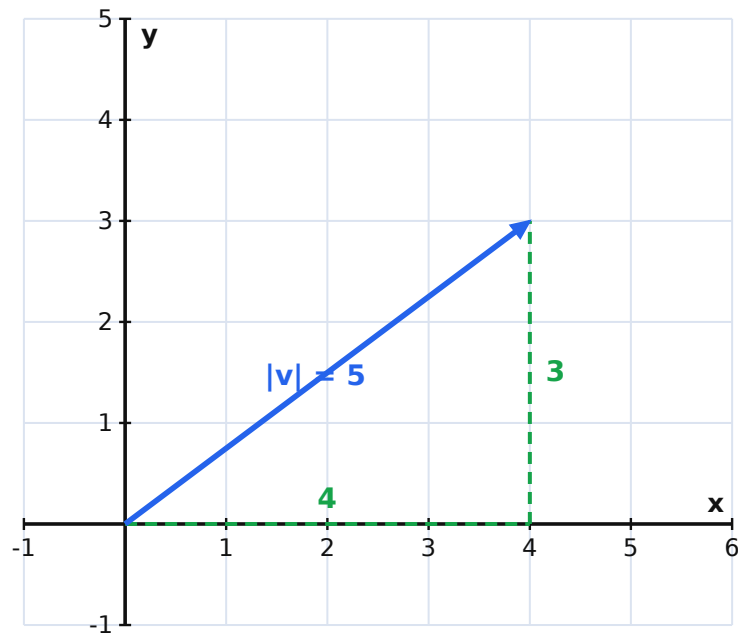
Precalculus · Common Core State Standards: **N-VM.A.1, N-VM.A.2, N-VM.A.3, N-VM.B.4, N-VM.B.5**

Key facts

- Vector from P to Q: $\langle x_2 - x_1, y_2 - y_1 \rangle$; magnitude $\sqrt{a^2 + b^2}$.
- Components of magnitude r at angle θ : .
- Unit vector in the direction of $\mathbf{v}$: $\mathbf{v}$ divided by $|\mathbf{v}|$.
- Dot product: $\langle a, b \rangle \cdot \langle c, d \rangle = ac + bd$; zero means perpendicular.
- Resultant force or velocity is the vector sum of the parts.

What is a vector?

A vector has both magnitude (length) and direction. In the plane, the vector from the origin to the point (a, b) is written in component form $\langle a, b \rangle$. The vector from $P(x_1, y_1)$ to $Q(x_2, y_2)$ is $\langle x_2 - x_1, y_2 - y_1 \rangle$. Its magnitude is $|\vec{v}| = \sqrt{a^2 + b^2}$, and its direction angle θ satisfies $\tan \theta = \frac{b}{a}$, measured from the positive x -axis (use the quadrant of $\langle a, b \rangle$).

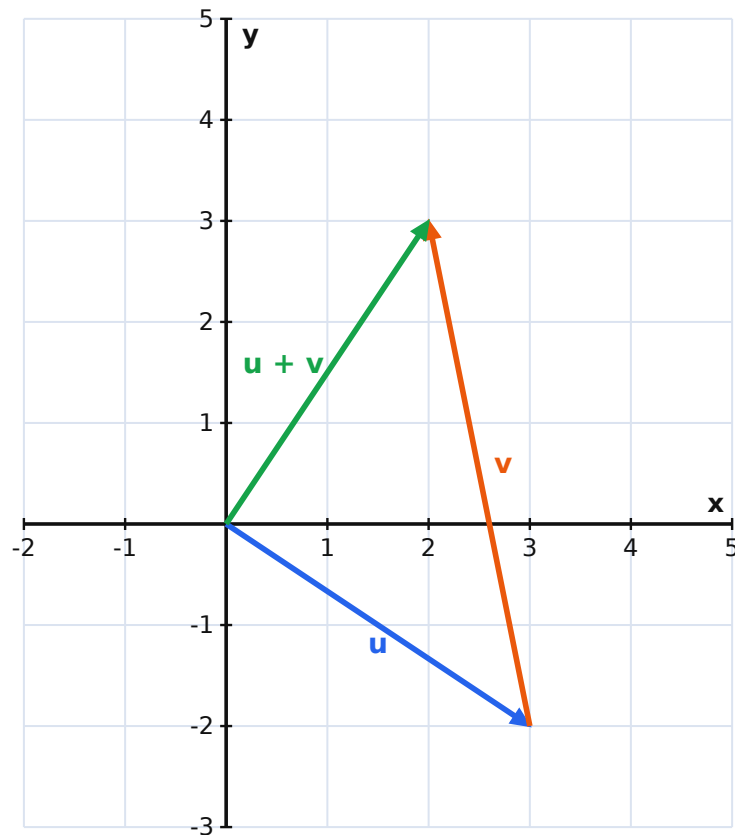


The vector $\langle 4, 3 \rangle$ has magnitude $\sqrt{16+9}=5$ and direction $\arctan\frac{3}{4} \approx 36.9^\circ$.

A vector with magnitude r and direction θ has components $\langle r\cos\theta, r\sin\theta \rangle$. A **unit vector** has magnitude 1. To find the unit vector in the direction of $\vec{v}$, divide by its magnitude: $\frac{\vec{v}}{|\vec{v}|}$. The standard unit vectors are $\vec{i} = \langle 1, 0 \rangle$ and $\vec{j} = \langle 0, 1 \rangle$, so $\langle a, b \rangle = a\vec{i} + b\vec{j}$.

How do you add and scale vectors?

Add or subtract component by component, and multiply a scalar by each component: $\langle 3, -2 \rangle + \langle -1, 5 \rangle = \langle 2, 3 \rangle$ and $3\langle 2, -1 \rangle = \langle 6, -3 \rangle$. Geometrically, place the tail of the second vector at the head of the first; the sum runs from the first tail to the second head.



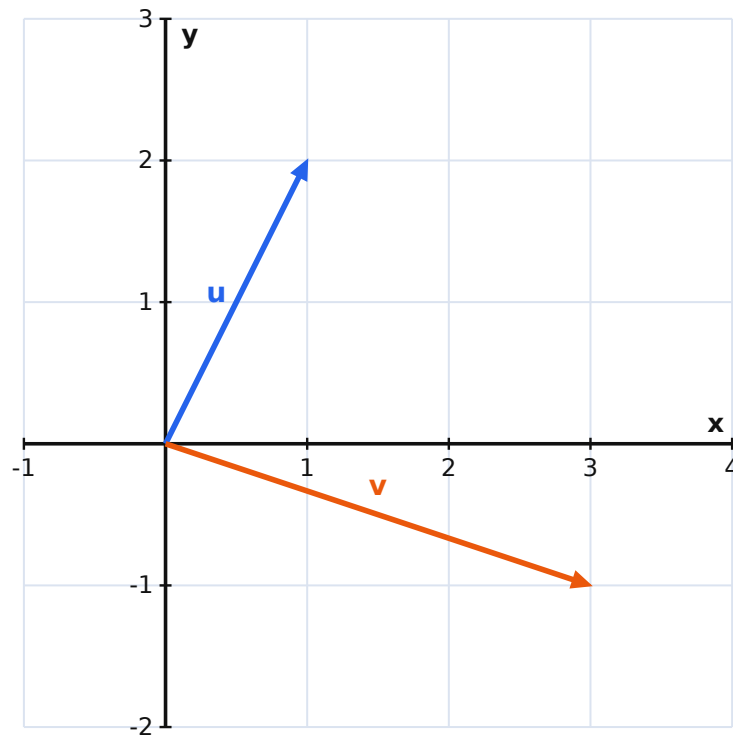
Head-to-tail addition gives the resultant vector.

What is the dot product?

Dot product

$\langle a, b \rangle \cdot \langle c, d \rangle = ac + bd$. Also $\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta$, where θ is the angle between the vectors.

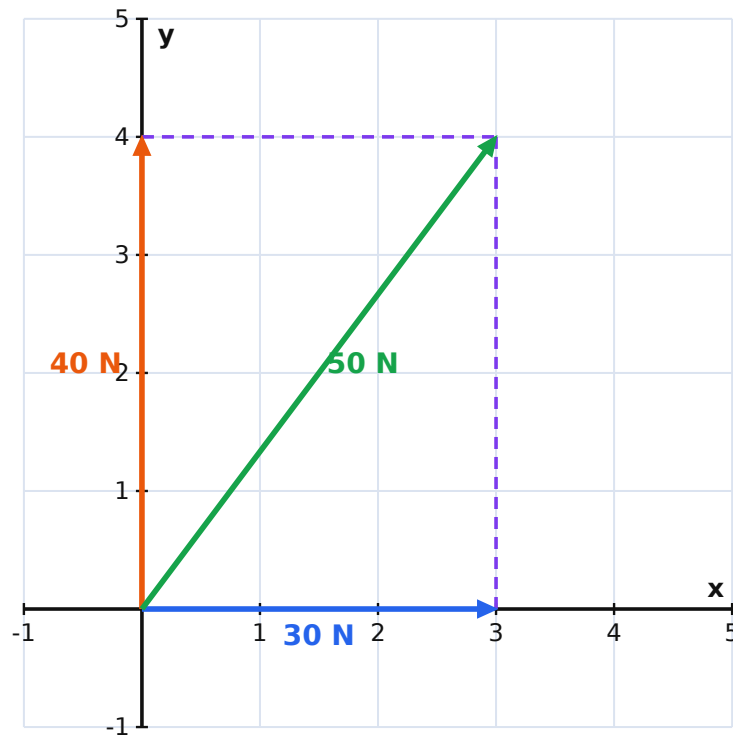
The dot product is a number, not a vector. Rearranging gives the angle: $\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|}$. Two vectors are perpendicular exactly when their dot product is 0. For $\vec{u} = \langle 1, 2 \rangle$ and $\vec{v} = \langle 3, -1 \rangle$: $\vec{u} \cdot \vec{v} = 1$, $|\vec{u}| = \sqrt{5}$, $|\vec{v}| = \sqrt{10}$, so $\cos \theta = \frac{1}{\sqrt{50}}$ and $\theta \approx 81.9^\circ$.



The angle between $\langle 1, 2 \rangle$ and $\langle 3, -1 \rangle$ is about 81.9° .

How are vectors used in applications?

Forces, velocities and displacements are vectors. When two forces act on an object, the resultant force is their vector sum. A 30 N force east and a 40 N force north combine to a 50 N force at 53.13° north of east. For a moving airplane, add the plane's velocity relative to the air and the wind's velocity to get the velocity relative to the ground. Work done by a constant force $\vec{F}$ over a displacement $\vec{d}$ is the dot product $W = \vec{F} \cdot \vec{d}$.



The resultant of two perpendicular forces.

Questions about vectors in the plane

What is the difference between a vector and a scalar?

A scalar is just a number, like 5 mph. A vector has a size and a direction, like 5 mph north, and is written with components.

How do you find the magnitude of a vector?

Use the Pythagorean theorem on its components: $|\langle a, b \rangle| = \sqrt{a^2 + b^2}$. For $\langle 5, -12 \rangle$ it is $\sqrt{25 + 144} = 13$.

What does the dot product tell you?

It measures how much two vectors point the same way. It is positive for angles under 90 degrees, zero at 90 degrees and negative above 90 degrees.

How do you find a unit vector?

Divide each component by the vector's magnitude. For $\langle 6, 8 \rangle$ the magnitude is 10, so the unit vector is $\langle 3/5, 4/5 \rangle$.

How do you combine an airplane's velocity with wind?

Write both as component vectors and add them. The sum is the velocity relative to the ground, and its magnitude is the ground speed.