

Precalculus - Vectors in the Plane - Answer Key

Common Core N-VM.A.1, N-VM.A.2, N-VM.A.3, N-VM.B.4, N-VM.B.5 · maths-pdf.fr/en/

Use this key to check each vector computation. Solutions show components, magnitudes and angles step by step, with final answers in bold.

Precalculus · Common Core State Standards: **N-VM.A.1, N-VM.A.2, N-VM.A.3, N-VM.B.4, N-VM.B.5**

Exercise 1 - Component form from two points

$$\langle 7-2, -1-3 \rangle.$$

$$\langle 5, -4 \rangle$$

Exercise 2 - Magnitude

$$\sqrt{25+144} = \sqrt{169}.$$

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Exercise 3 - Direction angle

$|v| = \sqrt{4+12} = 4$. The point is in quadrant II; the reference angle is $\arctan \frac{2\sqrt{3}}{2} = \arctan \sqrt{3} = 60^\circ$, so $\theta = 180^\circ - 60^\circ$.

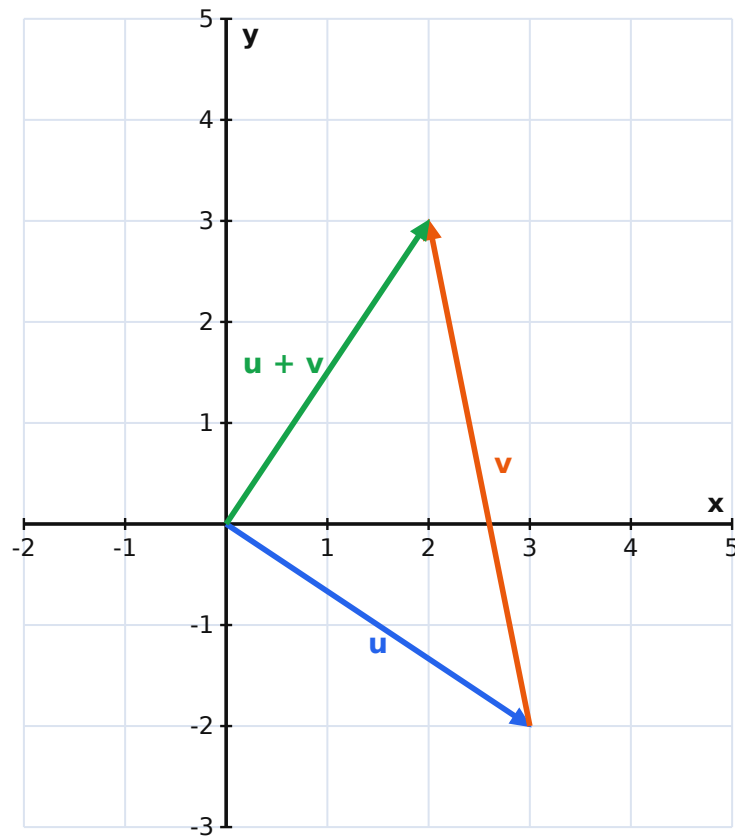
Magnitude 4; direction 120°

Exercise 4 - Components from magnitude and angle

$$\langle 10\cos 30^\circ, 10\sin 30^\circ \rangle = \langle 5\sqrt{3}, 5 \rangle \approx \langle 8.66, 5 \rangle.$$

$$\langle 5\sqrt{3}, 5 \rangle \approx \langle 8.66, 5 \rangle$$

Exercise 5 - Add vectors



The sum goes from the first tail to the last head.

$$\langle 3-1, -2+5 \rangle.$$

$$\langle 2, 3 \rangle$$

Exercise 6 - Subtract vectors

$$\langle 6-2, 4-7 \rangle.$$

$$\langle 4, -3 \rangle$$

Exercise 7 - Linear combination

$$\langle 6, -3 \rangle - \langle -2, 8 \rangle = \langle 8, -11 \rangle.$$

$$\langle 8, -11 \rangle$$

Exercise 8 - Unit vector

$|v|=10$. Divide each component by 10.

$$\left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$$

Exercise 9 - i and j form

$$4\vec{i} - 3\vec{j} \cdot \sqrt{16+9} = 5.$$

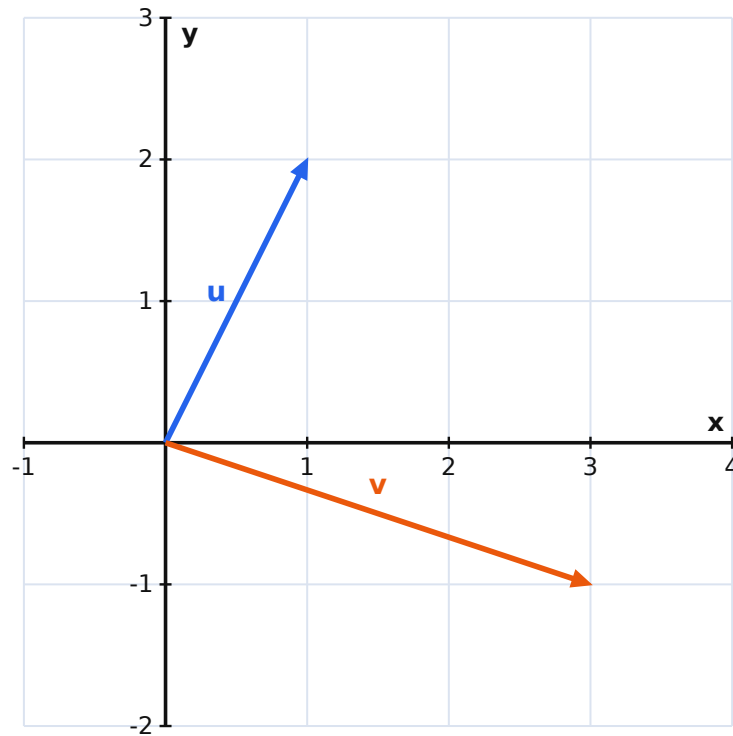
$$4\vec{i} - 3\vec{j}; 5$$

Exercise 10 - Dot product

$$2(-3) + 5(4) = -6 + 20.$$

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Exercise 11 - Angle between vectors



Angle of about 81.9 degrees.

$$\vec{u} \cdot \vec{v} = 3 - 2 = 1. |\vec{u}| = \sqrt{5}, |\vec{v}| = \sqrt{10}. \cos \theta = \frac{1}{\sqrt{50}} \approx 0.1414, \text{ so } \theta \approx 81.9^\circ.$$

81.9°

Exercise 12 - Perpendicular check

$$5(2) + (-2)(5) = 10 - 10 = 0.$$

Yes, the dot product is 0

Exercise 13 - Find k

The dot product must be zero: $8(3) + 12k = 0$, so $k = -2$.

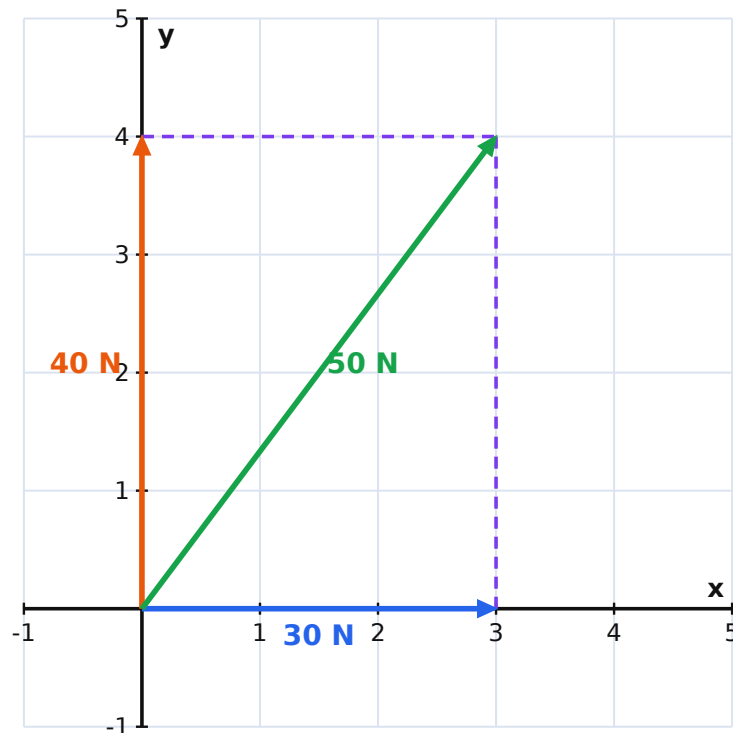
$$k = -2$$

Exercise 14 - Parallel vectors

$\langle -9, 6 \rangle = -3\langle 3, -2 \rangle$. One is a scalar multiple of the other, pointing the opposite way.

Parallel (opposite directions)

Exercise 15 - Two perpendicular forces



The resultant is the diagonal.

$\langle 30, 40 \rangle$: magnitude $\sqrt{900+1600}=50$. Direction $\arctan \frac{40}{30} \approx 53.13^\circ$.

50 N at 53.13° north of east

Exercise 16 - Airplane and wind

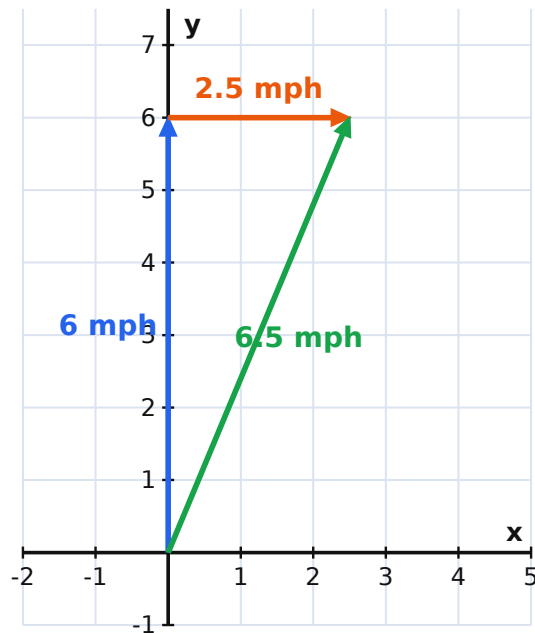


Ground velocity $\langle 40, 250 \rangle$.

Ground velocity $\langle 40, 250 \rangle$. Speed $\sqrt{1600+62500} \approx 253.2$. Angle east of north: $\arctan \frac{40}{250} \approx 9.1^\circ$.

253.2 mph; 9.1° east of north

Exercise 17 - Boat in a current



Speed 6.5 mph.

Speed $\sqrt{36+6.25}=6.5$. Drift $\arctan\frac{2.5}{6}\approx 22.62^\circ$.

6.5 mph; 22.62°

Exercise 18 - Work

$$30(6)+40(2)=180+80.$$

260 foot-pounds

Exercise 19 - Angle between two forces

$\cos\theta=\frac{15}{25}=0.6$, so $\theta\approx 53.13^\circ$. Resultant $\langle 8,4\rangle$: $\sqrt{64+16}=\sqrt{80}\approx 8.94$.

53.13° ; 8.94 N

Exercise 20 - Parallelogram

$\overrightarrow{AB}=\langle 3,1\rangle$ and $\overrightarrow{DC}=\langle 3,1\rangle$ are equal, so $ABCD$ is a parallelogram. $\overrightarrow{AC}=\langle 4,4\rangle$, $|\overrightarrow{AC}|=\sqrt{32}\approx 5.66$.

$$|\overrightarrow{AC}|=4\sqrt{2}\approx 5.66$$